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Systematic Singular Triangulations of All Orientable Seifert Manifolds

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1. Introduction

In this paper, we construct singular triangulations [1] of all orientable Seifert manifolds [2]. Especially, we consider singular triangulations with only one vertex, called *one-vertex triangulation*. Our construction is useful to calculate the state sum type invariant, for example, Turaev-Viro invariant, Turaev-Viro-Ocneanu invariant or Dijkgraaf-Witten invariant; this subject will be seen in forthcoming paper [3]. Also our work is made use of the introduction of a new complexity invariant of closed 3-manifold, see [4].

Let $\mathcal{M}$ be a Seifert manifold and P be a special spine [5] of $\mathcal{M}$. Considering a dual complex for $\mathcal{M}$ relative to P, we obtain a one-vertex triangulation of $\mathcal{M}$. Now, how to construct a special spine P of $\mathcal{M}$? Our construction is based on the fact that any orientable Seifert manifold is obtained by gluing M_n, J and $V_{p,q}$, which are homeomorphic to $(S^2 - \coprod_{i=1}^{n} D_i^2) \times S^1$, $(S^1 \times S^1 - D^2) \times S^1$ and (p, q)-type fibered solid torus respectively.

The first step is to make special spines P_{M_n}, P_J, $P_{V_{p,q}}$ of three compact manifolds M_n, J and $V_{p,q}$ satisfying the following conditions: each connected component of $\partial M_n \cap P_{M_n}$, $\partial J \cap P_J$ and $\partial V_{p,q} \cap P_{V_{p,q}}$ is the theta-curve shown in Figure 1 and the loop $\gamma\bar{\alpha}$ is a fiber, where $\bar{\alpha}$ means the reverse direction of the edge labeled α. As an example, the solid torus $V_{1,1}$

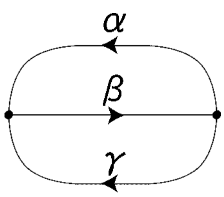

FIGURE 1. A theta-curve θ.

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