TOKYO J. MATH.
VOL. 28, NO. 2, 2005

A Characterization of Certain Einstein Kähler Hypersurfaces in a Complex Grassmann manifold of 2-planes

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(Communicated by Y. Ohnita)

1. Introduction

Denote by $G_r(\mathbf{C}^n)$ the complex Grassmann manifold of r-planes in $\mathbf{C}^n$, equipped with the Kähler metric of maximal holomorphic sectional curvature c.

One of the simplest typical examples of submanifolds of $G_r(\mathbf{C}^n)$ is a totally geodesic submanifold. B. Y. Chen and T. Nagano in [4, 5] determined maximal totally geodesic submanifolds of $G_2(\mathbf{C}^n)$. I. Satake and S. Ihara in [11, 6] determined all (equivariant) holomorphic, totally geodesic imbeddings of a symmetric domain into another symmetric domain. When an ambient symmetric domain is of type $(\mathrm{I})_{p,q}$, taking a compact dual symmetric space, we obtain the complete list of maximal totally geodesic Kähler submanifolds of $G_r(\mathbf{C}^n)$.

Let M be a maximal totally geodesic Kähler submanifold of $G_r(\mathbf{C}^n)$ given by a Kähler immersion $\varphi : M \to G_r(\mathbf{C}^n)$. Since M is a symmetric space, denote by (G, K) the compact symmetric pair of M. Then there exists a certain unitary representation $\rho : G \to \tilde{G} = SU(n)$, such that $\varphi(M)$ is given by the orbit of $\rho(G)$ through the origin in $G_r(\mathbf{C}^n)$.

Denote by $\mathbf{C}P^n$ and Q^n, an n-dimensional complex projective space and an n-dimensional complex quadric respectively.

EXAMPLE 1 ([4, 5, 11, 6]). Let $M = G/K$ be a proper maximal totally geodesic Kähler submanifold of $G_r(\mathbf{C}^n)$, ρ a corresponding unitary representation of G to $SU(n)$. Then, M and ρ are one of the following (up to isomorphism).

(1) $M_1 = G_r(\mathbf{C}^{n-1}) \hookrightarrow G_r(\mathbf{C}^n), \quad 1 \leqq r \leqq n-2$

(2) $M_2 = G_{r-1}(\mathbf{C}^{n-1}) \hookrightarrow G_r(\mathbf{C}^n), \quad 2 \leqq r \leqq n-1$

(3) $M_3 = G_{r_1}(\mathbf{C}^{n_1}) \times G_{r_2}(\mathbf{C}^{n_2}) \hookrightarrow G_{r_1+r_2}(\mathbf{C}^{n_1+n_2}), \quad 1 \leqq r_i \leqq n_i - 1,\ i = 1, 2$

(4) $M_4 = M_{4,p} = Sp(p)/U(p) \hookrightarrow G_p(\mathbf{C}^{2p}), \quad p \geqq 2$

(5) $M_5 = M_{5,p} = SO(2p)/U(p) \hookrightarrow G_p(\mathbf{C}^{2p}), \quad p \geqq 4$

(6) $M_{6,m} = \mathbf{C}P^p \hookrightarrow G_r(\mathbf{C}^n),\ r = \binom{p}{m-1},\ n = \binom{p+1}{m},\ 2 \leqq m \leqq p-1,$
$\rho_{6,m} : SU(p+1) \to SU(n)$: the exterior representation of degree m

Received March 2, 2004