TOKYO J. MATH.
VOL. 30, NO. 2, 2007

On Blanchard Manifolds

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Introduction

A compact complex manifold M of dimension 3 is called a *Blanchard manifold*, if its universal covering is biholomorphic to the complement of a projective line ℓ in a three dimensional complex projective space $\mathbf{P}^3$. Let Ω denote the complement $\mathbf{P}^3 \setminus \ell$. Then M is given as a quotient space Ω/Γ, where Γ is a group of holomorphic automorphisms of Ω. By [K, Theorem C], we know that

THEOREM A. (1) *The group Γ is a subgroup of the projective general linear group* $\mathrm{PGL}(4, \mathbf{C})$.

(2) *Γ contains a free abelian subgroup Γ_0 of finite index.*

(3) *By a suitable choice of homogeneous coordinates $[z_0 : z_1 : z_2 : z_3]$ on $\mathbf{P}^3$ with*

$$\ell = \{z_2 = z_3 = 0\},$$

Γ_0 is contained in either

$$\left\{\begin{pmatrix} 1 & a & b & c \\ 0 & 1 & a & b \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\} \qquad \text{Type (A)}, \tag{1}$$

or

$$\left\{\begin{pmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\} \qquad \text{Type (B)}. \tag{2}$$

When we say that Γ_0 is of type(A), $\mathrm{rank}(I-g) = 3$ for some $g \in \Gamma_0$. Otherwise, we say that Γ_0 is of type(B). It is known that, if Γ_0 of type(B), $\mathrm{rank}(I-g) = 2$ for any $g \in \Gamma_0 \setminus \{I\}$ ([K, Proposition 5.40]). In this short note we shall prove the following

Received February 17, 2005; revised February 26, 2007
Mathematics Subject Classification: 32H02, 32J17.
Key words: Blanchard manifold, flat projective structure, non-Kähler manifold.