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The Partial Vanishing of Victoria's Cohomology of Euclidean Superspaces

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1. Introduction

The notion of supermanifolds is introduced by Berezin–Leites([3]), but we should go back to Berezin's work on the mathematical formulation of the second quantization ([1]) when we talk about the origin of supermanifold theory. Berezin constructed Hamiltonians of relativistic (and also nonrelativistic) spin systems. Formulating spin systems is probably the first motivation of supermanifolds, and supermanifold theory is also used in describing supersymmetry with "superspace formulation" in quantum field theory (*cf.* [4]) in the present. We do not give the definition of supermanifolds in this paper. For the definition and elementary properties of supermanifolds, see [2], [3], [5].

All the results of this paper was announced in [6] without proof, and the purpose of this paper is to show the proofs of these results.

This paper and [6] are works on Victoria's cohomology ([7]) of Euclidean superspace (for the definition see [5]). Although we do not give the definition of Victoria's cohomology concretely, we review Victoria's cohomology shortly in the following.

We denote by $\mathbf{R}^{m|n}$ the $m|n$–dimensional Euclidean superspace. For a supermanifold M, we denote by $C^\infty(M)$ the superalgebra of superfunctions on M, and denote by $\mathfrak{X}(M)$ the $C^\infty(M)$–supermodule of vector fields on M (*i.e.*, $\mathfrak{X}(M)$ is the supermodule of superderivations on $C^\infty(M)$).

By definition, $\Omega^{1|0}(M) = \mathfrak{X}(M)^*$ ($\mathfrak{X}(M)^*$ denotes the $C^\infty(M)$–dual supermodule of $\mathfrak{X}(M)$) and $\Omega^{0|1}(M) = \Pi\mathfrak{X}(M)^*$ (Π is the parity changing functor).

Let $(x_1, \dots, x_{m+n}) = (u_1, \dots, u_m, \xi_1, \dots, \xi_n)$ denote a coordinate system of $\mathbf{R}^{m|n}$. $\mathfrak{X}(\mathbf{R}^{m|n})$ is a free $C^\infty(\mathbf{R}^{m|n})$–supermodule and $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_{m+n}}$ form a basis of $\mathfrak{X}(\mathbf{R}^{m|n})$. If we denote $\mathrm{d}^0 x_i := \left(\frac{\partial}{\partial x_i}\right)^*$ and $\mathrm{d}^1 x_i := \Pi\left(\frac{\partial}{\partial x_i}\right)^*$, clearly $\mathrm{d}^0 x_i, \dots, \mathrm{d}^0 x_{m+n}$ is a $C^\infty(\mathbf{R}^{m|n})$–basis of $\Omega^{1|0}(\mathbf{R}^{m|n})$ and $\mathrm{d}^1 x_1, \dots, \mathrm{d}^1 x_{m+n}$ is a $C^\infty(\mathbf{R}^{m|n})$–basis of $\Omega^{0|1}(\mathbf{R}^{m|n})$.

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