Seminear-rings Characterized by their $\mathscr{S}$-ideals. II

By Javed AHSAN

Department of Mathematical Sciences, King Fahd University of Petroleum and Minerals, Saudi Arabia

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This paper is a continuation of the author's earlier paper [1]. For undefined terms and notations used here we refer to [1]. In section 1 we describe some properties of the lattice of $\mathscr{S}$-ideals of a distributively generated SI-seminear-ring (cf. [1]). In section 2 we define a topology in the space of all prime $\mathscr{S}$-ideals in a distributively generated SI-seminear-ring, and show that the subset consisting of all minimal prime $\mathscr{S}$-ideals forms a Hausdorff space. Below we announce our results, whose details will appear elsewhere. Only some indications of proof will be given to Theorems 3, 4.

1. Distributively generated SI-seminear-rings. Throughout this section R will denote a d.g. seminear-ring with an absorbing zero as defined in [1]. As remarked in [1], the product AB of $\mathscr{S}$-ideals A and B of R is an $\mathscr{S}$-ideal. Moreover, for each family of $\mathscr{S}$-ideals $\{A_i : i \in I\}$ of R, the sum $\sum_{i\in I} A_i$ as defined in [1], is the unique minimal member of the family of all $\mathscr{S}$-ideals of R containing the $\mathscr{S}$-ideals $\{A_i : i \in I\}$; and $\cap_{i\in I} A_i$ is the unique maximal member of the family of all $\mathscr{S}$-ideals of R contained in the $\mathscr{S}$-ideals $\{A_i : i \in I\}$. Using these facts, we may state Propositions 2.2 and 2.3 given in [1] in the following forms.

Proposition 1. *The following assertions are equivalent*:

(1) *R is SI.*

(2) *For each pair of $\mathscr{S}$-ideals A, B of R, $A \cap B = AB$.*

(3) *The set of $\mathscr{S}$-ideals of R (ordered by inclusion) is a semilattice $(\mathscr{L}_R, \Lambda)$ with $A \Lambda B = AB$ for each pair of $\mathscr{S}$-ideals A, B of R.*

Proposition 2. *The following assertions are equivalent*:

(1) *R is SI.*

(2) *The set of all $\mathscr{S}$-ideals of R (ordered by inclusion) forms a complete lattice $\mathscr{L}_R$ under the sum and intersection of $\mathscr{S}$-ideals with $I \cap J = IJ$ for each pair of $\mathscr{S}$-ideals I, J of R.*

We also have:

Proposition 3. *The following assertions are equivalent*:

(1) *For each pair of $\mathscr{S}$-ideals A, B of R, $A \cap B = AB$.*

(2) *R is SI.*

(3) *For each pair of $\mathscr{S}$-ideals A, B of R, $B \cap A = AB$.*

(4) *For each pair of $\mathscr{S}$-ideals A, B of R, $A \cap (A^{-1}B) = A \cap B$ $(A^{-1}B = \{r \in R : ra \in B$ for all $a \in A\})$.*

Next we show that the lattice $\mathscr{L}_R$ described in Proposition 2, is a (complete) Brouwerian and hence distributive lattice. A lattice $\mathscr{L}$ is called *Brouwerian* if for any a, $b \in \mathscr{L}$, the set of all $x \in L$ satisfying $a \wedge x \leq b$ contains a greatest element c, the *pseudo-complement* of a relative to b.

Proposition 4. *If R is an SI-seminear-ring, then the lattice $\mathscr{L}_R$ is distributive.*

Analogous to the notion of prime ideals in near-ring theory ([2], p. 62), we call an $\mathscr{S}$-ideal P of a seminear-ring R *prime* if $IJ \subseteq P \Rightarrow I \subseteq P$ or $J \subseteq P$ holds for all $\mathscr{S}$-ideals I, J of R; P is called *completely prime* if for a, $b \in R$, $ab \in P \Rightarrow a \in P$ or $b \in P$; P *is minimal prime* if P is a minimal element of the set of prime $\mathscr{S}$-ideals of R. An $\mathscr{S}$-ideal K of R is *semiprime* if for all $\mathscr{S}$-ideals I of R, $I^2 \subseteq K \Rightarrow I \subseteq K$; K is *completely semiprime* if for $a \in R$ and n a positive integer, $a^n \in K \Rightarrow a \in K$. Furthermore, an $\mathscr{S}$-ideal Q of a seminear-ring R is called *irreducible* (*strongly irreducible*) if $I \cap J = Q \Rightarrow I = Q$ or $J = Q (I \cap J \subseteq Q \Rightarrow I \subseteq Q$ or $J \subseteq Q)$ holds for all $\mathscr{S}$-ideals I, J or R. Thus any prime $\mathscr{S}$-ideal is strongly irreducible and any strongly irreducible $\mathscr{S}$-ideal is irreducible. The following proposition shows that the concepts of prime, irreducible and strongly irreducible $\mathscr{S}$-ideals coincide for SI-seminear-rings.

Proposition 5. *Let R be an SI-seminear-ring. Then the following assertions for an $\mathscr{S}$-ideal P of R are equivalent*: