87. *Remark on Kohnen-Zagier's Paper Concerning Fourier Coefficients of Modular Forms of Half Intergral Weight*

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Introduction. In [5], Shimura has established a correspondence Ψ between the space of modular forms of half integral weight and the space of those of integral weight. Using the methods and languages of representation theory of adeles of metaplectic groups, Waldspurger [8] and [9] showed that the square of Fourier coefficients $a(n)$ for a square free integer n of the modular form $f(z) = \sum_{n=1}^{\infty} a(n)e[nz]$ of half integral weight is essentially proportional to the special value of the zeta function at a certain integer attached to the modular form F if f corresponds to F by Ψ and f is an eigen-function of Hecke operators.

On the other hand, Kohnen-Zagier [1] and [2] determined explicitly the proportion of the square of $a(n)$ for a square free integer n of $f(z) = \sum_{n=1}^{\infty} a(n)e[nz]$ which is an eigen-function of Hecke operators and belongs to the Kohnen's subspace $S^{+}_{(2k+1)/2}(4N) = \{f(z) = \sum_{(-1)^k n \equiv 0,1(4)} a(n)e[nz] \in S_{(2k+1)/2}(4N)\}$ of $S_{(2k+1)/2}(4N)$ by the special value of the zeta function associated with the modular form $F = \Psi(f)$ of integral weight. Kohnen-Zagier [1] (resp. Kohnen [2]) treated the case where $N = 1$ (resp. N is an odd square free integer). The purpose of this note is to derive an analogy of [1] and [2] in the case where f is an eigen-function of Hecke operators in $S_{(2k+1)/2}(4N)$ (resp. $S_{(2k+1)/2}(4N, \chi_N)$) and $\Psi(f)$ is a primitive form in $S_{2k}(2N)$, where $S_{(2k+1)/2}(4N)$ (resp. $S_{(2k+1)/2}(4N, \chi_N)$) means the vector space consisting of modular cusp forms of weight $(2k+1)/2$ and of level $4N$ (resp. of level $4N$ with the character χ_N). Since the modular form f satisfying our conditions is contained in orthogonal complement of Kohnen's space, there is no overlap between Kohnen-Zagier's results and ours. The method of our proof is similar to that of [1]. To prove our results, we need to modify their methods.

§1. Notation and preliminaries. We denote by Z, Q, R and C the ring of rational integers, the rational number field, the real number field and the complex number field, respectively. For $z \in C$, we put $e[z] = \exp(2\pi iz)$ and we define $\sqrt{z} = z^{1/2}$ so that $-\pi/2 < \arg(z^{1/2}) \leqq \pi/2$. Further, we put $z^{k/2} = (\sqrt{z})^k$ for every $k \in Z$. Let $SL(2, R)$ denote the group of all real matrices of degree 2 with determinant one and $\mathfrak{H}$ the complex upper half plane. For each positive integer N, put

$$\Gamma_0(N) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, R) \mid a, b, c \text{ and } d \in Z \text{ and } c \equiv 0 (\text{mod } N)\right\}.$$

We introduce an automorphic factor $j(\gamma, z)$ of $\Gamma_0(4)$ defined by $j(\gamma, z) =$