31. *On the Second Microlocalization along Isotropic Submanifolds*

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1. Introduction. Around 1985, Lebeau [9] developed the theory of the second microlocalization along isotropic submanifolds, and defined the "Γ-analytic microfunction" that had the unique continuation properties along bicharacteristic leaves. In this paper, we will give an explicit representation using the boundary values of holomorphic functions as Okada-Tose [10] did in the case of regular involutive submanifolds. As a consequence, we prove that "Γ-analytic microfunctions" form a strictly wider subclass of the microfunctions than that of the microfunctions with holomorphic parameters. Moreover, we can show the unique continuation properties of "Γ-analytic microfunctions" elementarily using the local version of Bochner's tube theorem [8]. Note that our methods of proofs are completely different from Lebeau's. We shall announce the results which will be proved in the subsequent paper [7].

2. Lebeau's second FBI-transformation. Let M be open in $\boldsymbol{R}^n_x = \boldsymbol{R}^d_{x'} \times \boldsymbol{R}^{n-d}_{x''}$, X be its complexification. We take coordinates of $T^*_M X \simeq \sqrt{-1}\, T^*M$ as $(x', x''; \sqrt{-1}\,\xi' dx' + \sqrt{-1}\,\xi'' dx'')$, and define its regular involutive submanifold Λ as $\{\xi' = 0\}$. Let $\dot{\xi}'' \in \boldsymbol{R}^{n-d}$ be a fixed non zero vector, and set

(2.1) $$\Gamma := \{x'' = 0,\ \xi' = 0,\ \xi'' = \dot{\xi}''\} \subset \Lambda \subset \sqrt{-1}\,T^*M.$$

Note that Γ is a bicharacteristic leaf of Λ. The following imbedding is intrinsically defined by Lebeau [9]

(2.2) $$\dot{T}^*\Gamma \ni (x'; \xi^{*\prime} dx') \mapsto (x' - \sqrt{-1}\,\xi^{*\prime}, 0) \in \boldsymbol{C}^d_{z'} \times \boldsymbol{C}^{n-d}_{z''} = \boldsymbol{C}^n_z = X.$$

Let $u(x)$ be a hyperfunction with compact support. We follow some definitions of Lebeau [9].

Definition 2.1 (Second FBI-transformation). We define the second FBI-transformation of u along Γ by

(2.3) $$T^2_\Gamma u(z; \lambda, \mu) := \int_{\boldsymbol{R}^n} u(x) \exp\left[-\frac{\lambda\mu^2}{2}(z' - x')^2 - \frac{\lambda}{2}(\mu z'' - x'')^2 - \sqrt{-1}\lambda x'' \cdot \dot{\xi}''\right] dx,$$

where $z = (z', z'') \in \boldsymbol{C}^d_{z'} \times \boldsymbol{C}^{n-d}_{z''}$ and $\lambda, \mu > 0$ are parameters.

Lebeau [9] has defined the second wave front set $S^2_\Gamma u$ of u as a closed subset of $\boldsymbol{C}^n$ in terms of $T^2_\Gamma u$ as follows.

Definition 2.2. For $z \in \boldsymbol{C}^n$, $z \notin S^2_\Gamma u$, if $\exists\, U$: a neighborhood of z in $\boldsymbol{C}^n$, $\exists\, \mu_0 > 0$, $\exists\, \delta > 0$, and

(2.4) $\exists\, f : (0, \mu_0) \to \boldsymbol{R}_+$ (a decreasing function) such that,