60. On Locally Trivial Families of Analytic Subvarieties with Locally Stable Parametrizations of Compact Complex Manifolds

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Introduction. The purpose of this note is to outline the recent results of our study on locally trivial families, i.e., families which locally are products, of *analytic subvarieties with locally stable parametrizations* of a compact complex manifold (cf. Definition 1.2 below), parametrized by (possibly nonreduced) complex spaces. The main theorem is as follows:

Main theorem. *Let Y be a compact complex manifold. We denote by $E(Y)$ the set of all analytic subvarieties with locally stable parametrizations of Y. We denote by Z_t an analytic subvariety with a locally stable parametrization of Y corresponding to a point $t \in E(Y)$. We define a subset $\tilde{\mathfrak{X}}(Y)$ of the product space $Y \times E(Y)$ by*

$$\tilde{\mathfrak{X}}(Y) := \{(y, t) \mid t \in E(Y), y \in Z_t\}.$$

We denote by $\tilde{\pi} : \tilde{\mathfrak{X}}(Y) \to E(Y)$ the restriction of the projection map Pr_{E_Y}: $Y \times E(Y) \to E(Y)$ to $\tilde{\mathfrak{X}}(Y)$. Then $E(Y)$ and $\tilde{\mathfrak{X}}(Y)$ have the structure of Hausdorff complex spaces which enjoy the following properties:

(i) *$\tilde{\mathfrak{X}}(Y)$ is a closed complex subspace of the product complex space $Y \times E(Y)$, and $\tilde{\pi} : \tilde{\mathfrak{X}}(Y) \to E(Y)$ is a locally trivial family of analytic subvarieties with locally stable parametrizations of Y, parametrized by $E(Y)$.*

(ii) (*Universality*) *Given a locally trivial family $\pi : \mathfrak{X} \to M$ of analytic subvarieties with locally stable parametrizations of Y, parametrized by a complex space M, there exists a unique holomorphic map $f : M \to E(Y)$ such that $f^*\tilde{\mathfrak{X}}(Y) = \mathfrak{X}$.*

(iii) *We denote by $D(Y)$ the Duady space of closed complex subspaces of Y, and by $\tilde{\pi}_0 : \tilde{\mathfrak{U}}(Y) \to D(Y)$ the universal family of closed complex subspaces of Y (cf.* [1]*). Then the inclusion map $\iota : E(Y) \to D(Y)$ is a holomorphic immersion and $\iota^*\tilde{\mathfrak{U}}(Y) = \tilde{\mathfrak{X}}(Y)$.*

(iv) (*C^∞ triviality*) *Let $t_0 \in E(Y)$ be a point whose corresponding point of $E(Y)_{red}$ (the reduction of $E(Y)$) is non-singular, then there exist an open neighborhood N of t_0 in $E(Y)$ and a diffeomorphism $\Psi : Y \times N \to Y \times N$ over N (i.e., $pr_N \circ \Psi = pr_N$) such that $\Psi(Z_{t_0} \times N) = \tilde{\pi}^{-1}(N)$.*

(v) (*C^∞ type constancy*) *Let t and t' be two points of the same connected component of $E(Y)$, then there exists a diffeomorphism $\varphi : Y \to Y$ such that $\varphi(Z_t) = Z_{t'}$.*

These results might be considered as a generalization of Namba's results in [6] to higher dimensional singular cases. Details will be published elsewhere.