53. *On Solutions of the Poincaré Equation*

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1. Introduction and result. Consider a map $F: \boldsymbol{C}^2 \to \boldsymbol{C}^2$ defined by

$$F: {}^t(x, y) \longmapsto {}^t(y, ax+p(y)), \tag{1}$$

where a is a nonzero constant and $p(y)$ is a polynomial of degree $d \geq 2$. The map F is called a *twisted elementary map* (Kimura [2]). We denote by F^k the k-times iteration of F. Assume that $z_0 = {}^t(x_0, y_0) \in \boldsymbol{C}^2$ is a periodic point of F of period k, i.e. a fixed point of F^k. Let J be the Jacobian matrix of F^k at z_0. Let ρ be an eigenvalue of J, $v = {}^t(v_1, v_2) \in \boldsymbol{C}^2$ an eigenvector of J corresponding to the eigenvalue ρ. The eigenvalue ρ is said to be *unstable* (resp. *stable*) if $|\rho| > 1$ (resp. if $|\rho| < 1$).

Definition (Kimura [2]). Suppose that ρ is unstable (resp. stable). A holomorphic map $\Xi: \boldsymbol{C} \to \boldsymbol{C}^2$ is called an unstable (resp. a stable) curve through z_0 if the following two conditions hold:

$$\Xi(\rho t) = F^k(\Xi(t)) \qquad \text{for } t \in \boldsymbol{C} \tag{2}$$

$$\Xi(t) = z_0 + vt + O(t^2) \qquad \text{as } t \longrightarrow 0. \tag{3}$$

If none of ρ^n $(n = 2, 3, 4, \cdots)$ is an eigenvalue of J, it is known that there exists an unstable (a stable) curve through z_0 ([2]). The functional equation (2) is called the *Poincaré equation*, since Poincaré [3] was the first to consider this type of functional equation (cf. Dixon-Esterle [1]). In this paper we shall establish the following:

Main theorem. *Each component of the* (*un*) *stable curve* $\Xi(t)$ *is an entire function of order* τ *and of finite type, where* τ *is given by*

$$\tau = \frac{\log d}{|\log |\rho|^{1/k}|}.$$

Remark. In a special case $k=1$, the result is already shown in [2]. As we shall see below, however, we require much subtler estimates than those in [2] to establish the theorem for $k>1$.

2. Notation. Throughout this paper we employ the following notation.

(a) Let $\Xi_m = {}^t(\xi_m, \eta_m): \boldsymbol{C} \to \boldsymbol{C}^2$ be holomorphic maps defined recursively by $\Xi_0(t) = \Xi(t)$ and

$$\Xi_m(t) = F(\Xi_{m-1}(\lambda^{-1}t)) \qquad \text{for } m \in \boldsymbol{Z}, \tag{4}$$

where $\lambda = \rho^{1/k}$. We put $\xi = {}^t(\xi_0, \cdots, \xi_{k-1})$ and $\eta = {}^t(\eta_0, \cdots, \eta_{k-1})$.

(b) For a k-vector $u = {}^t(u_0, \cdots, u_{k-1}) \in \boldsymbol{C}^k$, we put $\|u\| = |u_0| + \cdots + |u_{k-1}|$ and $p(u) = {}^t(p(u_0), \cdots, p(u_{k-1}))$.

(c) We put for $r > 0$,