# 49. *The Initial Boundary Value Problems for Linear Symmetric Hyperbolic Systems with Characteristic Boundary*

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**1. Introduction.** This paper studies the existence and differentiability of local solutions in time of the mixed initial boundary value problems for first order symmetric hyperbolic systems. We assume that the boundary is characteristic of constant multiplicity. A general theory for the case where the boundary is non-characteristic was developed by Friedrichs [2], Lax-Phillips [3] and Rauch-Massey III [7]. The case of the characteristic boundary has been treated by Lax-Phillips [3], Tsuji [9], Majda-Osher [5] and Rauch [6]. In particular, the tangential regularity of solutions for the latter case was obtained in [6].

The basic estimate we seek to establish is motivated by the work of Yanagisawa-Matsumura [10]. The norm used in that paper seems to be most suitable for our problem in the sense that "the loss of derivatives in the normal directions" is appropriately taken into account. (See also Chen [1].) Although we confine ourselves to the linear theory in this note, the main theorem is formulated in such a way that the applications to the quasilinear initial boundary value problems are possible.

Let $\Omega\subset\boldsymbol{R}^n$ be an open bounded set lying on one side of its boundary $\Gamma$. We treat differential operators of the form

$$L=A_0(v)\partial_t+\sum_{j=1}^{n}A_j(v)\partial_j+B(v),$$

where $\partial_t=\partial/\partial t$, $\partial_j=\partial/\partial x_j$, and $v=(v_1, v_2, \cdots, v_l)^t$ is a given smooth function of the time $t$ and the space variable $x=(x_1, x_2, \cdots, x_n)$. It is assumed that $A_0(\cdot)$, $A_j(\cdot)$, and $B(\cdot)$ are depending smoothly on their arguments. Therefore $A_0(v)$, $A_j(v)$, and $B(v)$ are smoothly varying real $l\times l$ matrices defined for $(t, x)\in[0, T]\times\bar{\Omega}$. We study the mixed initial boundary value problem

(1.1) $\quad Lu=F \qquad$ in $[0, T]\times\Omega$,

(1.2) $\quad M(x)u=0 \qquad$ in $[0, T]\times\Gamma$,

(1.3) $\quad u(0, x)=f(x) \qquad$ for $x\in\Omega$.

The unknown function $u(t, x)$ is a vector-valued function with $l$ components. $M(x)$ is an $l\times l$ real matrix depending smoothly on $x\in\Gamma$. $M$ is of constant rank everywhere. The inhomogeneous term $F$ of the equation and the initial data $f$ are given vector-valued functions defined on $[0, T]\times\bar{\Omega}$ and $\bar{\Omega}$, respectively. Let $\nu=(\nu_1, \nu_2, \cdots, \nu_n)$ be the unit outward normal to

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