98. *On Unit Groups of Algebraic Number Fields*

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1. Let K be a Galois extension of an algebraic number field k and U_K the unit group of the idele group $K_A^\times$. $O_K^\times$ denotes the global unit group of K. $N_{K/k}$ denotes the norm map from $K_A^\times$ to $k_A^\times$.

In our paper [2], we have proved the following isomorphism

$$(*)\qquad (N_{K/k}^{-1}(1)\cap(U_K\cdot K^\times))/(N_{K/k}^{-1}(1)\cap U_K)(N_{K/k}^{-1}(1)\cap K^\times)$$
$$\cong(O_k^\times\cap N_{K/k}K^\times)/N_{K/k}O_K^\times.$$

In this paper, this result will be generalized by using the cohomological language.

2. First, we consider the following commutative diagram of cochain complexes with exact rows and columns.

$$(1)\qquad \begin{tikzcd} & 0 \arrow[d] & 0 \arrow[d] & & \\ 0 \arrow[r] & A_1 \arrow[r, "a_1"] \arrow[d, "\varphi_1"] & A_2 \arrow[r, "a_2"] \arrow[d, "\varphi_2"] & A_3 \arrow[r] \arrow[d, equal] & 0 \\ 0 \arrow[r] & B_1 \arrow[r, "b_1"] \arrow[d, "\psi_1"] & B_2 \arrow[r, "b_2"] \arrow[d, "\psi_2"] & A_3 \arrow[r] & 0 \\ & C_1 \arrow[r, equal] \arrow[d] & C_1 \arrow[d] & & \\ & 0 & 0 & & \end{tikzcd}$$

Let us denote the connecting homomorphisms derived from (1) by

$$\delta_1: H^r(A_3)\longrightarrow H^{r+1}(A_1),$$
$$\delta_2: H^r(A_3)\longrightarrow H^{r+1}(B_1),$$
$$\gamma_1: H^r(C_1)\longrightarrow H^{r+1}(A_1),$$
$$\gamma_2: H^r(C_1)\longrightarrow H^{r+1}(A_2)\qquad (r\in \boldsymbol{Z}).$$

We denote the homomorphism $H^r(A_1)\to H^r(A_2)$ induced from a_1 by the same symbol a_1. The homomorphisms $a_2, b_1, b_2, \varphi_1, \varphi_2, \psi_1, \psi_2$ are defined in a similar way. Then, by the elementary diagram chasing, we have the following lemma.

Lemma. *With the notation as above, we have the following isomorphisms*

$$(2)\qquad H^r(B_2)/(\operatorname{Im}\varphi_2+\operatorname{Im}b_1)\cong\operatorname{Ker}a_1\cap\operatorname{Ker}\varphi_1\subset H^{r+1}(A_1),$$
$$(3)\qquad H^r(A_1)/(\operatorname{Im}\delta_1+\operatorname{Im}\gamma_1)\cong\operatorname{Ker}b_2\cap\operatorname{Ker}\psi_2\subset H^r(B_2).$$

Proof. Proof of (2). Since $\operatorname{Im}b_1=\operatorname{Ker}b_2$, we have

$$H^r(B_2)/(\operatorname{Im}\varphi_2+\operatorname{Im}b_1)\cong b_2(H^r(B_2))/b_2\varphi_2(H^r(A_2)).$$