59. *The Irreducible Decomposition of the Unramified Principal Series*

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1. Introduction. In what follows, we summarize some results on the irreducible decomposition of unramified principal series representations of quasi-split groups. A detailed account will be published elsewhere. In the case of split groups, the corresponding results were obtained by Rodier [3].

Let G be a connected reductive algebraic group defined over a non-archimedean local field F. We assume G is unramified, that is, G is quasi-split over F and split over an unramified extension of F. Let E be the minimal splitting field of G. Let S be a maximal F-split torus in G defined over F, T the centralizer of S in G, which is a maximal torus in G, and B a Borel subgroup of G defined over F containing T. We denote by $G(F)$, $B(F)$, $\cdots$, the locally compact and totally disconnected groups consisting of F-rational points of $G, B, \cdots$. Let $X^*(S)$ be the character group of S, $V=X^*(S)\otimes \boldsymbol{R}$ the vector space over the real number field $\boldsymbol{R}$ and Φ the relative root system of G with respect to S. A "root ray" of G with respect to S is an open half line with the starting point 0 in V containing at least one root relative to S. Let Ψ be the set of root rays of G with respect to S. For $a \in \Psi$, let $\sigma(a)$ (resp. $\tau(a)$) be the non-divisible (resp. non-multipliable) root contained in a. A root ray a is called *plural* if $\sigma(a)\neq\tau(a)$. We take the coroot system $\Psi^\vee$ attached to the reduced root system $\{\tau(a)\,|\,a\in\Psi\}$. The coroot corresponding to a root $\tau(a)$ is denoted by $a^\vee$. For $a\in\Psi$, we choose an absolute root α of G with respect to T such that the restriction of α to S equals $\sigma(a)$. Let Γ_α be the stabilizer of α in the Galois group Γ of E over F and $d(a)$ the index of Γ_α in Γ. Note that $d(a)$ is independent of the choice of α. Further, when a is a plural root ray, we put $\varepsilon(a)=(d(a)/2)+\pi(\log(q_F))^{-1}\sqrt{-1}$, where q_F is the cardinality of the residual field of F and $\pi=3.141\cdots$.

2. The unramified principal series. Let T_0 be the maximal compact subgroup of $T(F)$. An element of $X_0(T)=\mathrm{Hom}(T(F)/T_0, \boldsymbol{C}^*)$ is called an *unramified character* of $T(F)$. The relative Weyl group $W_G(S)$ corresponding to S acts on $X_0(T)$, namely, for $w\in W_G(S)$ and $x\in X_0(T)$, the action of w on χ is defined by $\chi^w(t)=\chi(\underline{w}^{-1}t\underline{w})$, $t\in T(F)$, where $\underline{w}$ is a representative of w in the group of F-rational points of the normalizer of S in G. An unramified character χ is called *regular* if $\chi^w\neq\chi$ for any $w\in W_G(S)$, $w\neq 1$. Let $X_{\mathrm{reg}}(T)$ be the set of regular unramified characters of $T(F)$. Note that