106. *Large Time Behavior of a Solution of a Parabolic Equation*

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In this paper, we shall prove that a solution of the following Cauchy problem converges to a constant as $t\to\infty$.

(1) $\partial_t u = Au + \sum_{|\alpha|=2q} B_\alpha(t,x)\partial^\alpha u, \quad t>0, \quad x\in \boldsymbol{R}^d; \quad u(0,x)=u_0(x),$

where

$$A\equiv(-1)^{q-1}\rho\sum_{k=1}^d \frac{\partial^{2q}}{\partial x_k^{2q}}$$

with a natural number q and a complex number ρ such that $\mathrm{Re}\,\rho>0$, $B_\alpha(t,x)$'s are in a class $\mathcal{F}^0(\boldsymbol{R}^+,\boldsymbol{R}^d)$ and "smaller" than $\mathrm{Re}\,\rho$, and $u_0(x)$ is in a class $\mathcal{F}^0(\boldsymbol{R}^d)$.

In case of the second order uniformly parabolic equation of the divergence structure, i.e. $\partial_t u=\sum_{j,k=1}^d \partial/\partial x_j(A_{jk}(t,x)\partial u/\partial x_k)$, many authors studied the behavior of the solution as $t\to\infty$ with the order of the convergence (for example see [1, 2]). However their proofs can not be applied to (1), and also in our case u_0 is not necessarily a function in $L_1(\boldsymbol{R}^d)$. Hence our assertion is proved based on the representation of the solution by *a Girsanov type formula* established in [3, 4].

1. For multi index α and $x\in\boldsymbol{R}^d$, we put

$$x^\alpha\equiv\prod_{k=1}^d x_k^{\alpha_k} \quad\text{and}\quad \partial^\alpha\equiv\prod_{k=1}\left(\frac{\partial}{\partial x_k}\right)^{\alpha_k}.$$

Give a non-negative number κ. $\mathcal{M}^\kappa(\boldsymbol{R}^d)$ is a Banach space consisting of all complex valued measures $\mu(d\xi)$ on $\boldsymbol{R}^d$ with

$$\|\mu\|_\kappa\equiv\int(1+|\xi|)^\kappa|\mu|(d\xi)<\infty,$$

and $\mathcal{F}^\kappa(\boldsymbol{R}^d)$ is a Banach space of all Fourier transforms of $\mathcal{M}^\kappa(\boldsymbol{R}^d)$, i.e.

$$f(x)=\int\exp\{i\xi\cdot x\}\mu_f(d\xi), \qquad \mu_f\in\mathcal{M}^\kappa(\boldsymbol{R}^d),$$

and we define as $\|f\|_\kappa\equiv\|\mu_f\|_\kappa$. $\mathcal{F}^0(\boldsymbol{R}^d)$ is a subset of uniformly continuous and bounded functions, $\sup_x|f(x)|\leqq\|f\|_0$, and the Schwartz class, $\sin\eta\cdot x$, constants, etc. are contained in $\mathcal{F}^\kappa(\boldsymbol{R}^d)$ for any $\kappa\geqq 0$.

Put $\boldsymbol{R}^+\equiv[0,\infty)$, and $\mathcal{M}^\kappa(\boldsymbol{R}^+,\boldsymbol{R}^d)$ denotes a set of all complex valued measures $\mu(t,d\xi)$, $t\in\boldsymbol{R}^+$, such that (i) $\mu\in\mathcal{M}^\kappa(\boldsymbol{R}^d)$ for each $t\in\boldsymbol{R}^+$, and (ii) $\|\mu(t,\cdot)-\mu(s,\cdot)\|_\kappa\to 0$ as $t\to s$ on $\boldsymbol{R}^+$. $\mathcal{F}^\kappa(\boldsymbol{R}^+,\boldsymbol{R}^d)$ is a Banach space consisting of all Fourier transforms of $\mathcal{M}^\kappa(\boldsymbol{R}^+,\boldsymbol{R}^d)$, i.e.

$$g(t,x)=\int\exp\{i\xi\cdot x\}\mu_g(t,d\xi), \qquad \mu_g\in\mathcal{M}^\kappa(\boldsymbol{R}^+,\boldsymbol{R}^d),$$

with a norm $\sup_{t\geqq 0}\|\mu_g(t,\cdot)\|_\kappa$. $\mu_g^*\in\mathcal{M}^\kappa(\boldsymbol{R}^d)$ is said *a dominating measure* of