24. *Path Integral for Some Systems of Partial Differential Equations*

By Hiroshi WATANABE
Department of Mathematics, Tokyo Metropolitan University

(Communicated by Kôsaku YOSIDA, M. J. A., March 12, 1984)

§ 1. **Introduction.** The Feynman path integral has been discussed by many authors and has various mathematical formulations (see [1]–[3] and the references cited in [2]). In each case we find a new Feynman-Kac formula extended to the quantum mechanical wave equations by generalizing the notion of measure. Albeverio and Höegh-Krohn [1] considered an analytic continuation of the characteristic function of the Wiener measure to integrate formally some functionals and gave fundamental solutions for Schrödinger equations in the form of the path integral.

In this note we shall propose a generalization of [1] to a wider class of operators which involves some hyperbolic systems and Schrödinger operators as special cases.

§ 2. **Formulation of the path integral.** We write $\partial=(\partial/\partial x_1, \cdots, \partial/\partial x_d)$, $D=-i\partial$, $D_\tau=-i\partial/\partial\tau$ and $\langle p\rangle=(1+|p|^2)^{1/2}$. We consider a free Hamiltonian $H(D)$ having the following properties: 1) The symbol $H(p)$ of the free Hamiltonian is an $r\times r$ matrix-valued continuous function on $\boldsymbol{R}^d$ satisfying

(2.1) $$|H(p)|<C\langle p\rangle^m, \qquad p\in \boldsymbol{R}^d,$$

for some constants $C>0$ and $m\geq 1$; and 2) for any $p\in\boldsymbol{R}^d$, $H(p)$ is a dissipative matrix, that is,

(2.2) $$\mathrm{Im}\,(H(p)X, X)>0, \qquad X\in \boldsymbol{C}^r,$$

where $(\cdot, \cdot)$ is the inner product in $\boldsymbol{C}^r$.

The Dirac operator $\sum_{j=1}^3 \alpha_j D_j+\mu\beta$ and the operators $-\Delta/2$ and $-i\Delta/2$, for example, have these properties with $m=1, 2$ and 2, respectively.

In the following we regard the Lebesgue space $L^m=L^m([s,t]\,;\boldsymbol{R}^d)$ as a measurable space whose σ-algebra is the set of all Borel sets in L^m. Put $m'=m/(m-1)$. We say that a map $\Psi(\cdot): L^{m'}([s,t]\,;\boldsymbol{R}^d)\to\boldsymbol{C}^r$ belongs to $F^m(s,t)$ if Ψ is a characteristic function of some $\boldsymbol{C}^r$-valued bounded measure $\hat{\Psi}$ on $L^m([s,t]\,;\boldsymbol{R}^d)$, that is,

$$\Psi(v)=\int_{L^m} e^{-i\langle v,\xi\rangle}\hat{\Psi}(d\xi), \qquad v\in L^{m'}([s,t]\,;\boldsymbol{R}^d),$$

where $\langle v,\xi\rangle$ denotes the pairing of $v\in L^{m'}$ and $\xi\in L^m$.

Let us introduce the symbol $\prod_s^\tau \exp[iH(\xi(\sigma))d\sigma]$, $\xi\in L^m([s,t]\,;\boldsymbol{R}^d)$,