

63. On the Spaces of Self Homotopy Equivalences of Certain CW Complexes

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1. Introduction. Let X be a connected CW complex with base point which is a vertex of X . And let $G(X)$ and $G_0(X)$ be the space of self homotopy equivalences of X with the compact open topology and the space (subspace of $G(X)$) of self homotopy equivalences of X preserving the base point, respectively. When X is an Eilenberg-MacLane complex $K(\pi, n)$, the weak homotopy type of $G(X)$ and $G_0(X)$ are completely determined by R. Thom [4] and D. H. Gottlieb [1], but it seems that little is known about the homotopy type of $G(X)$ and $G_0(X)$.

2. Results. Now, let X and Y be connected locally finite CW complexes with base points. Then there exists the following homeomorphisms (see [3]),

$$(X \times Y)^{X \times Y} \cong X^{X \times Y} \times Y^{X \times Y} \cong (X^X)^Y \times (Y^Y)^X,$$

$$(X \times Y)_0^{X \times Y} \cong X_0^{X \times Y} \times Y_0^{X \times Y} = (X^X, X_0^X)^{(Y, y_0)} \times (Y^Y, Y_0^Y)^{(X, x_0)},$$

where Z_0^K denotes the space of maps of K to Z preserving the base points with the compact open topology, $(Z, Z')^{(K, L)}$ denotes the space of maps of (K, L) to (Z, Z') and $(Z, Z')^{(K, L)}$ is regarded as a subspace of Z^K . Under these correspondences we have the following two theorems.

Theorem 1. *Let X and Y be connected locally finite CW complexes with base points. For given $n > 0$, assume that $\pi_i(X) = 0$ for every $i > n$ and $\pi_i(Y) = 0$ for every $i \leq n$. Then we have*

$$G(X \times Y) = G(X)^Y \times G(Y)^X,$$

$$G_0(X \times Y) = (G(X), G_0(X))^{(Y, y_0)} \times (G(Y), G_0(Y))^{(X, x_0)}.$$

Theorem 2. *For given $n > 0$, let X be a connected locally finite CW complex with base point whose dimension is not greater than n and let Y be an n -connected locally finite CW complex with base point. Then the same formulas on $G(X \times Y)$ and $G_0(X \times Y)$ as in Theorem 1 hold.*

These theorems are obtained by considering the induced homomorphisms of homotopy groups of self map of $(X \times Y, (x_0, y_0))$.

Let X be a connected locally finite CW complex with base point. Then every arcwise connected component of $G(X)$ has the same homotopy type. The same fact holds for $G_0(X)$. More generally, we have