51. *Factorization of Entire Solutions of Some Difference Equations*

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1. **Introduction.** A meromorphic function $h(x)$ is said to be *factored* if there are a meromorphic function $f(x)$ and an entire function $g(x)$ such that $h(x)=f(g(x))$. $h(x)$ is said to be *pseudo-prime* if every such factorization $h=f\circ g$ implies that either f is rational or g is a polynomial [4]. In this paper, we will consider factorization of solutions of the equation

$$(1.1)\qquad y(x+1)=P(y(x)),$$

where $P(w)$ is a polynomial of degree $p\geqq 2$:

$$(1.2)\qquad P(w)=a_pw^p+\cdots+a_1w+a_0,\quad a_p\neq 0,\quad p\geqq 2.$$

Any meromorphic solution $y(x)$ of (1.1) is transcendental and entire, unless it is a constant [6], [7]. If there is a number λ such that

$$(1.3)\quad \lambda=P(\lambda)\ \text{ and }\ P'(\lambda)=1,\quad P(w)=\lambda+(w-\lambda)+A_m(w-\lambda)^{m+1}+\cdots,$$

then the difference equation (1.1) possesses an entire solution $\phi_\lambda(x)$ which is expanded asymptotically

$$(1.3')\qquad \phi_\lambda(x)\sim\lambda+x^{-1/m}\sum\nolimits_{j+k\geqq 0}c_{jk}x^{-j/m}\left(\frac{\log x}{x}\right)^k$$

as x tends to ∞ through $D(R,\varepsilon)$:

$$(1.3'')\qquad D(R,\varepsilon)=\{|x|>R,\ |\arg x-\pi|<(\pi/2)-\varepsilon\}\cup\{\mathrm{Im}\,[xe^{-i\varepsilon}]>R\}\cup\{\mathrm{Im}\,[xe^{i\varepsilon}]<-R\}$$

where $\varepsilon>0$ is arbitrarily fixed, and $R(>0)$ depends on ε and c_{m0}, in which m is the integer in (1.3) ($A_m\neq 0$) [5], [7].

If there is a number λ such that

$$(1.4)\qquad \lambda=P(\lambda)\ \text{ and }\ |P'(\lambda)|>1,$$

then (1.1) possesses an entire solution $s_\lambda(x)$ which is expanded as

$$(1.4')\qquad s_\lambda(x)=\lambda+\sum\nolimits_{j=1}^{\infty}p_jb^{jx}=\psi_\lambda(b^x)\qquad (\text{we write } P'(\lambda) \text{ as b})$$

where

$$(1.4'')\qquad \psi_\lambda(t)=\lambda+\sum\nolimits_{j=1}^{\infty}p_jt^j$$

is an entire solution of the Schröder equation

$$(1.1')\qquad w(bt)=P(w(t)).$$

Further we have shown that any entire solution $y(x)$ of (1.1) satisfies

$$(1.5)\qquad y(x-\mu)\to\lambda\quad \text{as}\quad \mu\uparrow\infty,$$

uniformly on any compact set, where λ is a number for which either (1.3) or (1.4) holds [7], [8]. If $y(x)$ satisfies (1.5) for a λ with (1.3), then $y(x)$ can be written as