135. *Area Criteria for Functions to be Bloch, Normal, and Yosida*

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1. Introduction. Let $D=\{|z|<1\}$ and let

$$D(z,r)=\{w\in D\,;\,|w-z|/|1-\bar{z}w|<r\}$$

be the non-Euclidean disk of the non-Euclidean center $z\in D$ and the non-Euclidean radius $\tanh^{-1} r$, $0<r<1$. For f holomorphic in D we denote by $f(D(z,r))$ the image of $D(z,r)$ by f, namely, $f(D(z,r))$ is the set of w in the plane $C=\{|w|<\infty\}$ such that there exists $\zeta\in D(z,r)$ with $w=f(\zeta)$. Simply, $f(D(z,r))$ is the projection of the Riemannian image of $D(z,r)$ by f. Let $\alpha(z,r,f)$ be the Euclidean area of $f(D(z,r))$.

A prototype of our present study is

Theorem 1 [3]. *For f nonconstant and holomorphic in D to be Bloch, namely,*

$$\sup_{z\in D}(1-|z|^2)|f'(z)|<\infty,$$

it is necessary and sufficient that there exists r, $0<r<1$, such that

$$\sup_{z\in D}\alpha(z,r,f)<\infty.$$

We shall consider two natural analogues of Theorem 1 for normal meromorphic functions in D in the sense of O. Lehto and K. I. Virtanen [2], and for Yosida functions, namely, meromorphic functions (in the plane C) of K. Yosida's class (A) [4].

A function f meromorphic in D is said to be normal there if

$$\sup_{z\in D}(1-|z|^2)|f'(z)|/(1+|f(z)|^2)<\infty, \tag{1}$$

while a function f meromorphic in C is said to be Yosida if

$$\sup_{z\in C}|f'(z)|/(1+|f(z)|^2)<\infty. \tag{2}$$

For f meromorphic in D we let $\beta(z,r,f)$ be the spherical area of the image $f(D(z,r))$ of $D(z,r)$, $0<r<1$, contained in $C^*=C\cup\{\infty\}$, while, for f meromorphic in C we let $\gamma(z,r,f)$ be the spherical area of the image $f(\Delta(z,r))$ of the Euclidean disk $\Delta(z,r)=\{w\,;\,|w-z|<r\}$, $r>0$. Again, the images are the projections of the Riemannian images. Since C^*, regarded as the Riemann sphere of diameter one, has the spherical area π, we have two reasonable theorems, counterparts of Theorem 1.

Theorem 2. *For f nonconstant and meromorphic in D to be*