106. *Boolean Valued Analysis and Type I AW*-Algebras*

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1. Introduction. The structure theory of type I AW^*-algebras was instituted by Kaplansky [3] as a purely algebraic generalization of the theory of type I von Neumann algebras. However, his theory was not completed as he stated [3; p. 460], "One detail has resisted complete solution thus far: the uniqueness of the cardinal number attached to a homogeneous AW^*-algebra of type I." The above cardinal uniqueness problem has been open for 30 years (cf. [1; pp. 88, 111, 118 Exercise 10]) and Kaplansky [4; p. 843] conjectured that the answer is negative.

In this note, we shall outline a negative answer to this problem. Our method is due to Boolean valued analysis recently developed by Takeuti [7]–[9] and Ozawa [5], and the construction of the counterexample of the cardinal uniqueness problem will be reduced to P. J. Cohen's forcing argument (cf. [2], [10]) developed in the field of mathematical logic. Our argument also includes a complete classification of type I AW^*-algebras in terms of the cardinal numbers in Scott-Solovay's Boolean valued universe of set theory (cf. [10]). The proofs of the results in this note will be published in the forthcoming paper [6] with more detailed treatment. For the terminology and the basic theory of AW^*-algebras we shall refer to Berberian [1].

2. Boolean valued universe of sets. Let B be a complete Boolean algebra. *Scott-Solovay's Boolean valued universe* $V^{(B)}$ is defined by $V^{(B)}=\bigcup_{\alpha\in On} V_\alpha^{(B)}$, where $V_\alpha^{(B)}$ is defined by transfinite induction as follows: $V_0^{(B)}=\emptyset$ and

$$V_\alpha^{(B)}=\{u\,|\,u: \mathrm{dom}\,(u)\to B \quad \text{and} \quad \mathrm{dom}\,(u)\subseteq \textstyle\bigcup_{\beta<\alpha} V_\beta^{(B)}\}.$$

For any $u, v\in V^{(B)}$, the Boolean values $\|u\in v\|$ and $\|u=v\|$ are defined (cf. [10; § 13]), and then we define the Boolean value $\|\varphi(a_1,\cdots,a_n)\|$ for any formula φ of set theory with $a_1,\cdots,a_n\in V^{(B)}$ in the obvious way. There is a canonical embedding $u\to\check{u}$ of the universe V of sets into $V^{(B)}$ such that $\|\check{u}\in\check{v}\|$ ($\|\check{u}=\check{v}\|$) equals 1 if $u\in v$ ($u=v$) and equals 0 otherwise. The basic principles of Boolean valued analysis is the following transfer principle.

Theorem 1 (Scott-Solovay, cf. [10]). *If φ is a theorem of ZFC then $\|\varphi\|=1$ is also a theorem of ZFC.*

3. Hilbert spaces in $V^{(B)}$. We define real numbers as Dedekind