45. ∇-Poincaré's Lemma and ∇-de Rham Cohomology for an Integrable Connection with Irregular Singular Points

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Let M be a complex manifold and let H be a divisor on M. Denote by Ω^p the sheaf over M of germs of holomorphic p-forms and denote by $\Omega^p(*H)$ the sheaf over M of germs of meromorphic p-forms which are holomorphic in $M-H$ and have poles on H for $p=0,\cdots,n$. In case $p=0$, we use frequently $\mathcal{O}$ and $\mathcal{O}(*H)$ instead of Ω^0 and $\Omega^0(*H)$, respectively.

We suppose throughout this paper that the divisor H has at most normal crossings.

Let $\mathcal{S}$ be a locally free sheaf of $\mathcal{O}$-modules of rank m on M. For each point x in M, there exists a neighborhood U over which $S|_U$ is isomorphic to $(\mathcal{O}|_U)^m=\mathcal{O}\otimes_{\boldsymbol{C}}\boldsymbol{C}^m$. Denote the isomorphism by g_U. Define the locally free sheaf $\mathcal{S}\Omega^p(*H)$ of $\mathcal{O}(*H)$-modules of rank m over M by

$$\mathcal{S}\Omega^p(*H)=\mathcal{S}\otimes_{\mathcal{O}}\Omega^p(*H),$$

for $p=0,\cdots,n$. For $p=0$, instead of $\mathcal{S}\Omega^0(*H)$, we use frequently $\mathcal{S}(*H)$ of which the restriction to U, $\mathcal{S}(*H)|_U$ is isomorphic to $(\mathcal{O}(*H))^m|_U=(\mathcal{O}^m\otimes\mathcal{O}(*H))|_U$ by the mapping $g_U\otimes id$, which will be denoted also by g_U.

Let ∇ be a connection on $\mathcal{S}(*H)$: ∇ is an additive mapping of $\mathcal{S}(*H)$ into $\mathcal{S}(*H)\otimes_{\mathcal{O}(H^*)}\Omega^1(*H)=\mathcal{S}(*H)\otimes_{\mathcal{O}}\Omega^1=\mathcal{S}\otimes_{\mathcal{O}}\Omega^1(*H)=\mathcal{S}\Omega^1(*H)$ satisfying "Leibnitz rule"

$$\nabla(f\cdot u)=u\otimes df+f\cdot\nabla(u)$$

for all sections $f\in\mathcal{O}(*H)(U)$, $u\in\mathcal{S}\Omega^1(*H)(U)$. We suppose that the connection is integrable, that is, the composite mapping

$$\nabla^2:\mathcal{S}(*H)\longrightarrow\mathcal{S}\Omega^1(*H)\longrightarrow\mathcal{S}\Omega^2(*H)$$

is a zero mapping.

If we take adequately an open covering $\{U_k\}$ on M, then to give connection ∇ means the following; for each U_k, the mapping

$$g_{U_k}\circ\nabla\circ g_{U_k}^{-1}:(\mathcal{O}(*H)|_{U_k})^m\longrightarrow(\mathcal{O}(*H))^m\otimes_{\mathcal{O}}\Omega^1|_{U_k},$$

is induced by a mapping

$$\nabla_k:(\mathcal{O}(*H)(U_k))^m\longrightarrow((\mathcal{O}(*H)\otimes_{\mathcal{O}}\Omega^1)(U_k))^m,$$

which is represented by $(d+\Omega_k)$ under a generator system $\langle e_{k,1},\cdots,e_{k,m}\rangle$