106. Smooth Global Solutions for the One-Dimensional Equations in Magnetohydrodynamics

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§1. **Introduction.** The motion of electrically conducting fluids on one space coordinate is described by the equations in Lagrangian representation ([1]):

$$(1)\quad \begin{cases} (1/\rho)_t - u_x = 0, \\ u_t + (p+|B|^2/2\mu_0)_x = (\nu\rho u_x)_x, \qquad v_t - (\bar{B}^1 B/\mu_0)_x = (\mu\rho v_x)_x, \\ \theta_t + (\theta p_\theta/e_\theta)u_x = (1/e_\theta)\{(\kappa\rho\theta_x)_x + \nu\rho u_x^2 + \mu\rho|v_x|^2 + (\rho/\sigma\mu_0^2)|B_x|^2\}, \\ (B/\rho)_t - (\bar{B}^1 v)_x = \{(\rho/\sigma\mu_0)B_x\}_x. \end{cases}$$

Here $\rho>0$, $\boldsymbol{u}=(u^1,u^2,u^3)\in \boldsymbol{R}^3$, $\theta>0$ and $\boldsymbol{B}=(\bar{B}^1,B^2,B^3)\in \boldsymbol{R}^3$ represent the mass density, the velocity, the absolute temperature and the magnetic induction, where we write $u=u^1$, $v=(u^2,u^3)$, $B=(B^2,B^3)$, and $\bar{B}^1$ is a constant.

We assume that the pressure p and the internal energy e are smoothly related to ρ and θ by the equations of state

$$(2)\qquad p_\rho>0,\quad e_\theta>0,\quad de=\theta dS - pd(1/\rho),$$

where $S=S(\rho,\theta)$ is the entropy; the coefficients of viscosity μ,ν, the coefficient of heat conductivity κ and the coefficient of electrical resistivity $1/\sigma$ (σ: the coefficient of electrical conductivity) are all smooth functions of ρ and θ, and are positive or identically zero; μ_0 is the magnetic permeability, now a positive constant.

In this paper, we seek smooth solutions of (1) in a small neighborhood of a constant state $(\rho,\boldsymbol{u},\theta,B)=(\bar{\rho},0,\bar{\theta},\bar{B})$ where $\bar{\rho}>0$, $\bar{\theta}>0$ and $\bar{B}\in\boldsymbol{R}^2$ are arbitrary fixed constants. To obtain the a priori estimates for the solutions, we use the following energy form ([4]):

$$\mathcal{E}=e-\bar{e}+\bar{p}(1/\rho-1/\bar{\rho})-\bar{\theta}(S-\bar{S})+|\boldsymbol{u}|^2/2+|B-\bar{B}|^2/2\mu_0\rho,$$

where $\bar{e}=e(\bar{\rho},\bar{\theta})$ and so on. Note that if $|\rho-\bar{\rho},\theta-\bar{\theta}|$ is small, $\mathcal{E}$ is equivalent to the quadratic form $|\rho-\bar{\rho},\boldsymbol{u},\theta-\bar{\theta},B-\bar{B}|^2$. This is based on the strict convexity of the internal energy e as a function of $1/\rho$ and S.

From (1) and (2), we have the energy conservation law:

$$(3)\qquad (e+|\boldsymbol{u}|^2/2+|B|^2/2\mu_0\rho)_t+\{(p+|B|^2/2\mu_0)u-(\bar{B}^1B/\mu_0)\cdot v\}_x \\ =\{\nu\rho uu_x+\mu\rho v\cdot v_x+\kappa\rho\theta_x+(\rho/\sigma\mu_0^2)B\cdot B_x\}_x,$$

and the equation of entropy:

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