92. On 2-Rank of the Ideal Class Groups of Totally Real Number Fields

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§1. **Introduction.** We are concerned with the problem to construct infinitely many number fields of a given degree m and with a given number of real (resp. complex) absolute values r_1 (resp. r_2), for which the ideal class group contains a given finite abelian group A as a subgroup. Ishida [4] (resp. [5]) solved this problem when m is any odd prime number, $r_1=1$ (resp. $r_1=3$) and A is an elementary 2-abelian group with rank $m-1$ (resp. A is any elementary m-abelian group). But when $r_2=0$, no results are known to the author, except when m is small, i.e. when $m=2$ and A is cyclic (by Yamamoto [9] and Weinberger [8]) and when $m=3$ and A is cyclic (by Uchida [7] and Ichimura [3]).

In this paper, we consider the problem in the case $r_2=0$ and A is an elementary 2-abelian group. When m is even, this can be solved for any such A by composing a totally real number field of degree $m/2$ with a real quadratic field with a large genus number. When m is odd, we use the method of [4] to prove the following

Theorem. *For any odd natural number m (>1), there exist infinitely many totally real number fields of degree m, for which the ideal class group contains an elementary 2-abelian group with rank $(m-1)/2$ as a subgroup.*

Our method of the proof is sketched as follows. Let $f(X)=X\prod_{i=1}^{m-1}(X-A_i)-C^2$ be an irreducible polynomial, where A_i and C are rational integers satisfying some congruence and other conditions. Let θ be a root of $f(X)$, and set $K=\boldsymbol{Q}(\theta)$. Then, K is totally real and $K(\sqrt{\theta-A_1}, \sqrt{\theta-A_2}, \cdots, \sqrt{\theta-A_{m-1}})$ contains an unramified abelian extension over K of type $(2, \cdots, 2)$ with rank $(m-1)/2$.

Remark 1. Recently, Azuhata and Ichimura [1] solved our problem for any $r_1\geqq 0$, $r_2>0$ and any abelian group A with rank $\leqq r_2$. As in [1], we can solve the problem for any odd rational integer $r_1\geqq 1$, any rational integer $r_2\geqq 0$, and an elementary 2-abelian group A with rank $2r_2+(r_1-1)/2$.

§2. **Proof of the theorem.** Let m (>1) be a given odd number. We consider a polynomial of the form $f(X)=X\prod_{i=1}^{m-1}(X-\mathrm{A}_i)-C^2$ for rational integers A_i and C. Let p_i ($1\leqq i\leqq m-1$) be prime numbers