83. On 4-Manifolds Fibered by Tori

By Yukio MATSUMOTO
Department of Mathematics, University of Tokyo

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 13, 1982)

§ 1. **Definitions.** The class of elliptic surfaces plays an important role in the theory of complex surfaces, [2]. In this note, we consider an analogous structure on smooth 4-manifolds, which we call *torus fibration*, and announce some results. Before giving the definition of torus fibration, we need slightly extend the notion of fibered link in the 3-sphere.

Definition. A smooth map $g: S^3 \to \boldsymbol{C}$ is called a *multiple fibered link* if it satisfies the following:

(i) $g^{-1}(0) \neq \phi$;

(ii) the map $\varphi(\boldsymbol{x}) = g(\boldsymbol{x})/|g(\boldsymbol{x})| : S^3 - g^{-1}(0) \to S^1$ is a submersion;

(iii) at each $\boldsymbol{x}_0 \in g^{-1}(0)$, there exist local coordinates u_1, u_2, u_3 in S^3 so that

$$g(\boldsymbol{x}) = (u_2(\boldsymbol{x}) + \sqrt{-1}u_3(\boldsymbol{x}))^m$$

holds for all $\boldsymbol{x}$ near $\boldsymbol{x}_0$, m being a certain positive integer (called the *multiplicity at* $\boldsymbol{x}_0$).

Definition. A map $f: \boldsymbol{R}^4 \to \boldsymbol{C}$ is a *cone-extension* of a smooth map $g: S^3 \to \boldsymbol{C}$, if it is given as follows:

$$f(\boldsymbol{x}) = \begin{cases} \|\boldsymbol{x}\|^d g(\boldsymbol{x}/\|\boldsymbol{x}\|) & \boldsymbol{x} \neq \boldsymbol{0} \\ 0 & \boldsymbol{x} = \boldsymbol{0} \end{cases}$$

where d is an integer >0 depending on f.

Clearly f is smooth outside of the origin $\boldsymbol{0} \in \boldsymbol{R}^4$. Let $f_i: M_i^m \to N_i^k$ be a map, $p_i \in M_i^m$ a point, for $i=1,2$, where M_i^m and N_i^k are oriented smooth manifolds. We say that the germ (f_1, p_1) is *smoothly* (+)-*equivalent* to the germ (f_2, p_2) if they are equivalent through orientation preserving local diffeomorphisms around p_i and $f_i(p_i)$.

Now we define the torus fibration. Let M and B be oriented smooth manifolds of dimensions 4 and 2, respectively. In this note, we assume that M and B are closed for the sake of convenience.

Definition. *A torus fibration of M with base space B* is an onto map $f: M \to B$ with the following properties:

(i) at each point $p \in M$, the germ (f, p) is smoothly (+)-equivalent to the germ at $\boldsymbol{0}$ of a cone-extension of a multiple fibered link;

(ii) the inverse image $C_u = f^{-1}(u)$ of any *general* point $u \in B$ is diffeomorphic to the 2-torus T^2.

Note that the projection map f is smooth outside a finite set of