57. Conformally Related Product Riemannian Manifolds

By Yoshihiro TASHIRO and In-Bae KIM
Department of Mathematics, Hiroshima University

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Introduction. Let M and M^* be product Riemannian manifolds of dimension $n\geq 3$, and denote the structures by (M, g, F) and (M^*, g^*, G) respectively. The product structures F of M and G of M^* are different from the identity tensor I and satisfy the relations

$$F^2=I, \qquad G^2=I,$$
$$g(FX, FX)=g(X, X), \qquad g^*(GX^*, GX^*)=g^*(X^*, X^*)$$

for any vector X of M and any vector X^* of M^*. The integrability conditions of F and G in M and M^* are

$$\nabla_X F=0, \qquad \nabla^*_{X^*}G=0,$$

where we have denoted by ∇ and ∇^* covariant differentiations with respect to g and g^* respectively. A conformal diffeomorphism f of M to M^* is characterized by the change

$$g^*=\rho^{-2}g \tag{0.1}$$

of the metric tensors, where ρ is a positive valued scalar field.

Under a diffeomorphism f of M to M^*, the image of a quantity on M^* by the induced map f^* of f will be denoted by the same character as the original. The structures F and G are said to be *commutative* with one another at a point P of M under f if $FG=GF$ at P. In a previous paper [2], one of the present authors has proved the following

Theorem A. *If both product Riemannian manifolds M and M^* are complete, then there is no global non-homothetic conformal diffeomorphism of M to M^* such that the product structures F and G are not commutative under it at a point of M.*

As the contraposition of Theorem A, we can state

Theorem B. *Let both product Riemannian manifolds M and M^* be complete. If there exists a global non-homothetic conformal diffeomorphism f of M onto M^*, then the diffeomorphism f has to make the product structures F and G commutative everywhere in M.*

By virtue of Theorem B, we shall investigate product Riemannian manifolds admitting a global non-homothetic conformal diffeomorphism. The purpose of the present paper is to prove the following

Theorem. *Let both M and M^* be complete, connected and simply connected product Riemannian manifolds of dimension $n\geq 3$. If there is a global non-homothetic conformal diffeomorphism f of M onto M^*,*