# 40. A Perturbation Theory for Abstract Evolution Equations of Second Order

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**1. Introduction.** Let $X$ be a Banach space with norm $\|\cdot\|$ and $B(Y,X)$ be the totality of bounded linear operators mapping $Y$ into $X$. A subset $C(t)$, $t \in R$ of $B(X,X)$ is called a strongly continuous cosine family in $X$ if

(1) $C(t+s)+C(t-s)=2C(t)C(s)$ for all $t, s \in R$;

(2) $C(0)=I$;

(3) $C(t)x$ is continuous in $t \in R$ for each fixed $x \in X$.

The associated sine family is given by

$$S(t)x=\int_0^t C(r)x\,dr$$

for $x \in X$ and $t \in R$. The infinitesimal generator is the operator $A: D(A) \to X$ defined by $Ax=\lim_{h\to 0} 2h^{-2}(C(h)-I)x$ for $x \in D(A)$, where $D(A)=\{x \in X : \lim_{h\to 0} h^{-2}(C(h)-I)x \text{ exists}\}$. It is well known that for $\lambda>\omega$, $\lambda^2$ belongs to the resolvent set of $A$ and for $x \in X$

$$\lambda(\lambda^2-A)^{-1}x=\int_0^\infty e^{-\lambda t}C(t)x\,dt, \tag{4}$$

where $\omega$ is a constant with $\omega \geqq \log_e(1+2\|C(1)\|)$. (See [1, p. 90].)

The cosine family in $X$ with generator $A$ is associated with the Cauchy problem for the abstract evolution equation of second order in $X$

$$d^2u/dt^2=Au, \quad t \in R; \quad u(0)=u, \quad u'(0)=x. \tag{5}$$

It is natural to try to convert (5) into a well-posed first order system

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix}=\begin{pmatrix} 0 & I \\ A & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}, \quad t \in R; \quad \begin{pmatrix} u \\ v \end{pmatrix}(0)=\begin{pmatrix} u \\ x \end{pmatrix} \tag{6}$$

and to make use of the extensive theory of groups. (See, for example [6].)

For a strongly continuous cosine family $C(t)$, $t \in R$ in $X$ with the infinitesimal generator $A$, we are concerned with the set

$$E=\{x \in X ; C(t)x \text{ is once continuously differentiable in } t \in R\}.$$

Kisyński [2] proved the important facts that the set $E$ under the norm

$$|u|_E=\|u\|+\operatorname{Max}\{\|C'(s)u\| : 0 \leqq s \leqq 1\}$$

becomes a Banach space and that (5) can always be converted into the well-posed problem (6) in the Banach space $E \times X$.

In order to make this conversion more convenient we will, in this