102. *Analytic Hypo-Ellipticity of a System of Microdifferential Equations with Non-Involutory Characteristics*

By Toshinori ÔAKU
Department of Mathematics, University of Tokyo

(Communicated by Kôsaku YOSIDA, M. J. A., Nov. 12, 1981)

We study the analytic hypo-ellipticity of a system of microdifferential equations whose characteristic variety (in the complex domain) has the form $V=V_1\cup V_2$; here V_1 and V_2 are regular involutory complex submanifolds with non-involutory intersection. We also assume that the system has regular singularities along V (cf. [4]). In particular, the system $(P_1P_2I_m+A)u=0$ satisfies the above conditions if P_1 and P_2 are scalar operators such that the Poisson bracket $\{\sigma(P_1), \sigma(P_2)\}$ does not vanish (where σ denotes the principal symbol), A is an $m\times m$ matrix of operators of lower order, and I_m is the unit matrix of degree m (see Corollary in § 1).

Our result (Theorem in § 1) extends a part of the results of Kashiwara-Kawai-Oshima [3] to more general systems. We believe that our result is new even for single equations (see Example 2). The operator discussed in Corollary is contained in the class discussed by Treves [8] if $\sigma(P_2)$ is the complex conjugate of $\sigma(P_1)$. See also Grušin [1] for a class of single partial differential equations.

§ 1. Statement of the results. Let M be an n-dimensional real analytic manifold and X be its complexification. We denote by $\mathcal{C}_M$ the sheaf on T^*_MX of microfunctions, and by $\mathcal{E}_X$ the sheaf on T^*X of microdifferential operators of finite order. Let $\mathcal{M}$ be a system of microdifferential equations (i.e. a coherent $\mathcal{E}_X$-module) defined on an open subset Ω of T^*X-X. Suppose that the characteristic variety of $\mathcal{M}$ has the form $V=V_1\cup V_2\subset\Omega$. We assume the following conditions (see [4] for notations):

(A.1) V_1 and V_2 are d-codimensional homogeneous regular involutory submanifolds of Ω, and $V_0=V_1\cap V_2$ is non-singular.

(A.2) V_1 and V_2 intersect normally, i.e., $T_pV_1\cap T_pV_2=T_pV_0$ for any $p\in V_0$.

(A.3) $\dim V_1=\dim V_2=\dim V_0+1$.

(A.4) $\operatorname{rank} V_1=\operatorname{rank} V_2=\operatorname{rank} V_0$.

(A.5) $\mathcal{M}$ has regular singularities along V.

Let p_0 be a point of $V_0\cap T^*_MX$. We can find a neighborhood $\Omega'\subset\Omega$ of p_0 and a coherent sub-$\mathcal{E}_V$-module $\mathcal{M}_0$ of $\mathcal{M}|_{\Omega'}$ such that $\mathcal{E}_X\mathcal{M}_0=\mathcal{M}|_{\Omega'}$.