79. A Remark on the Hadamard Variational Formula. II

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§ 1. **Introduction.** Let $f(x)$ be a real-valued C^∞-function of x in R^n. Let $\Omega_t=\{x\in\mathrm{R}^n \mid f(x)<t\}$ for any real t. Then its boundary is $\gamma_t=\{x\in\mathrm{R}^n \mid f(x)=t\}$. We assume the following assumptions for f:

(A.1) Ω_2 is a bounded domain diffeomorphic to the unit disc.

(A.2) All values $t\in[-2,0)\cup(0,2]$ are regular values of f.

(A.3) Ω_2 contains only one critical point x^0 of f, where $f(x^0)=0$ and f has the non-degenerate Hessian of the index $n-1$.

For any $t\in[-1,0)\cup(0,1]$, we consider the following boundary value problem for u:

$$(\lambda-\Delta)u(x)=w(x), \qquad \text{for } x\in\Omega_t, \tag{1.1}$$

$$\frac{\partial}{\partial\nu}u(x)=0, \qquad \text{for } x\in\gamma_t, \tag{1.2}$$

where ν is the outer unit normal to γ_t and $\lambda\in\mathrm{C}$. If $\lambda>0$, u is uniquely determined by w and we put $u(x)=N_t(\lambda)w(x)$. Let $N_t(\lambda,x,y)$ be the integral kernel function of the mapping: $w\mapsto N_t(\lambda)w$, i.e.,

$$N_t(\lambda)w(x)=\int_{\Omega_t} N_t(\lambda,x,y)w(y)dy. \tag{1.3}$$

It is well known from the Hadamard variational formula that the function $N_t(\lambda,x,y)$ is continuously differentiable with respect to t if $t\neq 0$ and $x,y\in\Omega_{-1}$. The Hadamard variational formula implies that

$$\begin{aligned}\frac{d}{dt}N_t(\lambda,x,y) &= \int_{\gamma_t} N_t(\lambda,z,y)N_t(\lambda,z,x)\frac{1}{|\mathrm{grad}\, f(z)|}d\sigma(z)\\ &\quad+\int_{\gamma_t}\langle\nabla'_z N_t(\lambda,z,y),\ \nabla'_z N_t(\lambda,z,x)\rangle\frac{1}{|\mathrm{grad}\, f(z)|}d\sigma(z)\end{aligned} \tag{1.4}$$

where $d\sigma$ is the volume element of γ_t, $\nabla'_z N_t(\lambda,z,y)$ denotes the component tangent to γ_t of the gradient vector of $N_t(\lambda,z,y)$ with respect to z and $\langle\ ,\ \rangle$ denotes the inner product in the tangent vector space to γ_t. See, for instance, Hadamard [6], Aomoto [1], Peetre [8] and Fujiwara-Ozawa [3].

For any small $\varepsilon>0$, we have

$$N_1(\lambda,x,y)-N_\varepsilon(\lambda,x,y)=\int_\varepsilon^1\frac{d}{d\tau}N_\tau(\lambda,x,y)d\tau \tag{1.5}$$