63. A Generalized Poincaré Series Associated to a Hecke Algebra of a Finite or p-Adic Chevalley Group[*)]

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Introduction. Let (W, S) be a Coxeter system ([1]) with finite generator system S. The Poincaré series of W is by definition the formal power series $\sum_{w\in W} t^{l(w)}$, in which t is a variable and $l(w)$ is the length of w with respect to the generator system S of W. This series has arisen in works of many authors (see the references of [4]). Our main purpose is to investigate the properties of the formal power series of matrix coefficients $L(t, R)=L(t, q, W, R)$ defined by ($\sharp$) in § 1 for a representation R of the Hecke algebra H_q $(q>0)$ (see § 1 for the definition of H_q). (Note that if $q=1$ and R is trivial, $L(t, R)$ is just the Poincaré series (W, S).) In particular we show that $L(t, R)$ is similar, in property, to the congruence zeta function of an algebraic variety. See 1)–3) below. The original motivation of this work was to associate a kind of L-function to an irreducible representation of the Hecke algebra H_q (hence, to an irreducible constituent of the natural representation of G on the space of functions on G/B, where G is a finite (resp. p-adic) Chevalley group and B is a Borel (resp. Iwahori) subgroup of G). The main results of this paper are:

1) Components of $L(t, R)$ are rational functions (Theorem 1),

2) if W is finite,

i) the function $L(t, R)$ satisfies a functional equation (Theorem 2. (1)),

ii) the absolute values of the zeros of det $L(t, R)$ are of the forms q^{-a} for some rational numbers $0\leq a\leq 1$ (Theorem 2. (2)),

iii) the zeros on the boundary of 'the critical strip' can be described explicitly in terms of vertices of W-graph ([3]), if R has a W-graph (Theorem 3).

(The author can prove that any finite dimensional representation of a finite irreducible Coxeter group has a W-graph with the possible exception of the Coxeter group of type H_4. The details will be published elsewhere.)

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