43. *Zeta Functions in Several Variables Associated with Prehomogeneous Vector Spaces. III*[*)]

Eisenstein Series for Indefinite Quadratic Forms

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In the present note, by applying the general theory developed in [2], we prove functional equations of Eisenstein series for indefinite quadratic forms.

6. Let Y be an $n+1$ by $n+1$ rational non-degenerate symmetric matrix of signature (p, q) $(p+q=n+1)$. Denote by $d_i(A)$ the determinant of the upper left i by i block of a matrix A. Let Γ_∞ be the group of upper triangular integral matrices of size $n+1$ with diagonal entries 1. For an $n+1$ tuple $\varepsilon=(\varepsilon_1, \cdots, \varepsilon_{n+1})$ of ± 1, we write $\operatorname{sgn}\varepsilon=(i, n-i+1)$ if exactly i of ε_j's are equal to 1. For any $\varepsilon\in\{\pm 1\}^{n+1}$ with $\operatorname{sgn}\varepsilon=(p, q)$, the Eisenstein series for Y is defined by

$$E(Y, \varepsilon; s)=\sum_U \prod_{i=1}^n |d_i({}^tUYU)|^{-s_i} \quad (s=(s_1, \cdots, s_n)\in \boldsymbol{C}^n)$$

where U runs through a set of all representatives of the double cosets belonging to $SO(Y)_{\boldsymbol{Z}}\backslash SL(n+1)_{\boldsymbol{Z}}/\Gamma_\infty$ such that

$$d_i({}^tUYU)/|d_i({}^tUYU)|=\varepsilon_1\cdots\varepsilon_i \quad (1\leqq i\leqq n+1).$$

Let $z=(z_1, \cdots, z_{n+1})$ be a variable which is connected to s by $s_i=z_{i+1}-z_i+1/2$ $(1\leqq i\leqq n)$. Set

$$\Lambda(Y, \varepsilon; z)=\sum_{1\leqq j<i\leqq n+1} \eta(2z_i-2z_j+1)|\det Y|^{z_{n+1}}E(Y, \varepsilon; s)$$

where $\eta(z)=\pi^{-z/2}\Gamma(z/2)\zeta(z)$ ($\zeta(z)$: the Riemann zeta function).

Theorem 6. (1) *The series* $E(Y, \varepsilon; s)$ $(\varepsilon\in\{\pm 1\}^{n+1}, \operatorname{sgn}\varepsilon=(p, q))$ *are absolutely convergent for* $\operatorname{Re} s_1, \cdots, \operatorname{Re} s_n>1$.

(2) *The functions* $E(Y, \varepsilon; s)$ *multiplied by*

$$\prod_{1\leqq i\leqq j\leqq n}\left(s_i+s_{i+1}+\cdots+s_j-\frac{j-i}{2}-1\right)^2\zeta(2(s_i+s_{i+1}+\cdots+s_j)-j+i)$$

have analytic continuations to entire functions of s *in* $\boldsymbol{C}^n$.

(3) *For any permutation* σ *in* $n+1$ *letters and for any* ε, $\eta\in\{\pm 1\}^{n+1}$ *such that* $\operatorname{sgn}\varepsilon=\operatorname{sgn}\eta=(p, q)$, *there exists* $A^\sigma(\varepsilon, \eta; z)$ *a rational function of trigonometric functions of* z *satisfying*

$$\Lambda(Y, \varepsilon; \sigma z)=\sum_\eta A^\sigma(\varepsilon, \eta; z)\Lambda(Y, \eta; z)$$

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