87. Characteristic Cauchy Problems and Solutions of Formal Power Series

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§1. **Introduction.** Let C^{n+1} be the $(n+1)$-dimensional complex space. $z=(z_0, z')=(z_0, z_1, \cdots, z_n)$ denotes its point and $\xi=(\xi_0, \xi')=(\xi_0, \xi_1, \cdots, \xi_n)$ denotes its dual variable. We shall make use of the notation $\partial_z=(\partial_{z_0}, \partial_{z'})=(\partial_{z_0}, \partial_{z_1}, \cdots, \partial_{z_n})$, $\partial_{z_i}=\partial/\partial z_i$. For a linear partial differential operator $a(z, \partial_z)$, $a(z, \xi)$ denotes its total symbol. Now let us consider Cauchy problem in a neighbourhood Ω of $z=0$,

$$\text{(C.P)} \qquad \begin{cases} L(z, \partial_z)u(z)=((\partial_{z_0})^k-A(z, \partial_z))u(z)=f(z), \\ (\partial_{z_0})^i u(0, z')=\hat{u}_i(z'), \qquad 0\leq i\leq k-1, \end{cases}$$

where

$$A(z, \partial_z)=\sum_{i=0}^{k-1} A_i(z, \partial_{z'})(\partial_{z_0})^i \tag{1.1}$$

and $A(z, \partial_z)$ is an operator of order m and its coefficients and $f(z)$ are holomorphic in Ω and $\hat{u}_i(z')$ $(0\leq i\leq k-1)$ are holomorphic in $\Omega'=\Omega\cap\{z_0=0\}$. We can easily find out a solution of formal power series $\hat{u}(z)$ of (C.P) of the form

$$\hat{u}(z)=\sum_{n=0}^{\infty} \hat{u}_n(z')(z_0)^n/n!. \tag{1.2}$$

$\hat{u}_n(z')$ $(n\geq k)$ are successively and uniquely determined from (C.P). It follows from well-known Cauchy-Kovalevskaja theorem that whenever $m\leq k$, $\hat{u}(z)$ converges and is a unique holomorphic solution of (C.P).

The purpose of this paper is to give an anlytical interpretation of $\hat{u}(z)$, that is, existence of a solution $u_S(z)$ of the equation $L(z, \partial_z)u_S(z)=f(z)$ with the asymptotic expansion $\hat{u}(z)$ in a sector S, when $m>k$. So we assume $m>k$ in the following.

§2. **Characteristic indices.** In §2 we introduce a new notation, characteristic indices. Let us write $A(z, \partial_z)$ in the form different from (1.1),

$$A(z, \partial_z)=\sum_{i=0}^{m}\left(\sum_{l=s_i}^{i} a_{i,l}(z, \partial_{z'})(\partial_{z_0})^{i-l}\right), \tag{2.1}$$

where $a_{i,l}(z, \xi')$ is a homogeneous polynomial of ξ' with degree l and if $a_{i,l}(z, \xi')\equiv 0$ for all l, we put $s_i=+\infty$. We expand $a_{i,l}(z, \xi')$ at $z_0=0$ with respect to z_0,

$$a_{i,l}(z, \xi')=\sum_{j=0}^{\infty} a_{i,l,j}(z', \xi')(z_0)^j. \tag{2.2}$$

Put