47. Further Results for the Solutions of Certain Third Order Differential Equations

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1. Introduction. This paper is concerned with the differential equations of the form

(1.1) $\dddot{x}+a(t)f(x,\dot{x})\ddot{x}+b(t)g(x,\dot{x})+c(t)h(x)=p(t)+\tilde{p}(t,x,\dot{x},\ddot{x})$,

(1.2) $\dddot{x}+a(t)f(x,\dot{x})\ddot{x}+b(t)g(x,\dot{x})+c(t)h(x)=p(t)$

where $a, b, c, p, \tilde{p}, f, g, h$ are real valued functions.

The asymptotic property of solutions of third order differential equations has received a considerable amount of attention over the past two decades (cf. [1]–[8]). Many of these results are summarized in [9].

In [5], the author considered (1.1) in the case $p(t)\equiv 0$ and eatablished sufficient conditions under which all solutions of (1.1) and their first and second order derivatives are uniformly bounded and tend to zero as $t\to\infty$.

In Theorem 3.1 of this paper, sufficient conditions are given for uniform boundedness and convergence to zero of all solutions of (1.1) together with their derivatives of the first and second order. Theorem 3.1 generalizes our former result in [5]. In Theorem 3.2, necessary and sufficient conditions are given for uniform boundedness and convergence to zero of all solutions of (1.1) together with their derivatives of the first and second order.

2. Definition and lemma. Let us consider the following system

(2.1) $$\dot{x}=F(t,x)$$

where $F(t,x)$ is a continuous function from $[0,\infty)\times R^n$ to R^n. We denote the solution of (2.1) through (t_0, x_0) by $x(t, t_0, x_0)$.

Definition 2.1. The solutions of (2.1) are *uniformly bounded*, if for any $\alpha>0$, there exists $\beta(\alpha)>0$ such that

$$\|x(t,t_0,x_0)\|<\beta \qquad \text{for } \|x_0\|<\alpha \text{ and } t\geqq t_0\geqq 0.$$

For the proof of Theorems given below we need the following Lemma ([5, Theorem A]).

Lemma 2.1. *Suppose that there exists a Liapunov function $V(t,x)$, continuously differentiable in $[0,\infty)\times R^n$, satisfying the following conditions:*

(i) *$a(\|x\|)\leqq V(t,x)\leqq b(\|x\|)$, where $a(r), b(r)$ are continuous, increasing and positive definite functions and $a(r)\to\infty$ as $r\to\infty$.*