17. *Studies on Holonomic Quantum Fields. XII*

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In our previous note [1] we have considered a classical scattering problem for 2-dimensional massless Dirac fields, and characterized the "τ-function" $\langle g\otimes g^{-1}\rangle$ of the corresponding Clifford group element. As we shall see in this article, this procedure works in the Minkowski space-time $X^{Min}=\boldsymbol{R}^s$ of an arbitrary dimensionality s.

To put the matter somewhat differently, what we do amounts to calculate the following path integrals (or more precisely their product $\tau[A]\tau^*[A]$) in a closed form (see § 1):

$$(1)\qquad \tau[A]=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS_0+iS_{int}}\Big/\int \mathcal{D}\bar{\psi}\mathcal{D}\psi^{iS_0}=\langle \boldsymbol{T}(e^{iS_{int}})\rangle$$

$$\tau^*[A]=\int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-iS_0+iS_{int}}\Big/\int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-iS_0}=\langle \boldsymbol{T}^*(e^{iS_{int}})\rangle$$

$$S_0=\int d^sx\bar{\psi}(x)(i\partial\!\!\!/-m)\psi(x)$$

$$S_{int}=-\int d^sx\bar{\psi}(x)A\!\!\!/(x)\psi(x).$$

Here $A(x)=(A_\mu(x))$ is a given classical external field. Thus $\log \tau[A]$, when incorporated with the free action, gives the effective action for the "gauge field" $A(x)$. (The integral (1) is formally given by $\det(i\partial\!\!\!/-A\!\!\!/-m)/\det(i\partial\!\!\!/-m)$; however the meaning of an infinite dimensional determinant is obscure and should be made precise.)

Indeed we infer that the time-ordered (resp. anti time-ordered) product $\varphi[A]=\boldsymbol{T}(e^{iS_{int}})$ (resp. $\varphi^*[A]=\boldsymbol{T}^*(e^{iS_{int}})$) is nothing but the element of the Clifford group which induces the rotation $T[A]$ (resp. $T[A]^{-1}$), the classical scattering operator. To see this observe that

$$(2)\qquad (i\partial\!\!\!/-A\!\!\!/(x)-m)\boldsymbol{T}(e^{iS_{int}}\psi(x))=0$$

$$\boldsymbol{T}(e^{iS_{int}}\bar{\psi}(x))(i\overleftarrow{\partial\!\!\!/}+A\!\!\!/(x)+m)=0.$$

An arbitrary matrix element $w(x)=\langle\Phi_1|\boldsymbol{T}(e^{iS_{int}}\psi(x))|\Phi_2\rangle$ or $\overline{w}(x)=\langle\Phi_1|\boldsymbol{T}(e^{iS_{int}}\bar{\psi}(x))|\Phi_2\rangle$ satisfies the same equation (2), respectively. Now in the remote past or future we have

$$(3)\qquad w(x)\sim w_{in}(x)=\langle\Phi_1|\varphi[A]\psi(x)|\Phi_2\rangle \qquad (x^0\to-\infty)$$

$$w_{out}(x)=\langle\Phi_1|\psi(x)\varphi[A]|\Phi_2\rangle \qquad (x^0\to+\infty)$$

$$\overline{w}(x)\sim\overline{w}_{in}(x)=\langle\Phi_1|\varphi[A]\bar{\psi}(x)|\Phi_2\rangle \qquad (x^0\to-\infty)$$

$$\overline{w}_{out}(x)=\langle\Phi_1|\bar{\psi}(x)\varphi[A]|\Phi_2\rangle \qquad (x^0\to+\infty).$$

Along with the definition of $T[A]$, $(\overline{w}_{out}, w_{out})=T[A](\overline{w}_{in}, w_{in})$, (3) shows