80. P-Convexity with Respect to Differential Operators which act on Linear Subspaces

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§ 1. **Introduction.** We treat linear partial differential operators with constant coefficients of order m:

$$P=P(D),\ D=(D_1, \cdots, D_n),\ D_j=\frac{1}{i}\frac{\partial}{\partial x_j},\ P_m(D) \text{ is its principal part.}$$

We consider the problem of characterizing geometrically the open sets which are P-convex in the case P is independent of some of the variables.

Generally an open set Ω in R^n is called P-convex if for every compact set K in Ω, there exists a compact set K' in Ω such that for every u in $\mathcal{E}'(\Omega)$

$$\operatorname{supp} P(-D)u \subset K \quad \text{implies} \quad \operatorname{supp} u \subset K'.$$

The importance of this concept lies in the following property of P-convex sets proved by B. Malgrange [3]: The equation $P(D)u=f$ in Ω has a C^∞ solution u for every f in $C^\infty(\Omega)$ if and only if Ω is P-convex.

It is well known that an open convex set is P-convex for every differential operator P. However, complete characterizations of P-convexity are known only in the following cases:

1) P is elliptic (L. Hörmander [1]),
2) $n=2$ (L. Hörmander [1]),
3) P is of first order (E. C. Zachmanoglou [10]),
4) $n=3$, P is of principal type, i.e. $P_m(\xi)=0$ implies grad $P_m(\xi) \neq 0$ and $\partial\Omega$ is C^2 (J. Persson [8]),
5) $P(D)=D_1D_2+\frac{1}{2}\sum_{j=3}^{n} D_j$, and $\partial\Omega$ is C^2 (J. Persson [9]).

We restrict our attention to the operators as above, and we consider them as operators in R^{n+k}. Consequently, they are independent of the variables $(x_{n+1}, \cdots, x_{n+k})$.

Under these somewhat restricted situations, we obtain a sufficient condition for P-convexity with respect to such operators. Especially, in the case 1), we show that the sufficient condition is also necessary.

In § 2, we shall give the definition and the properties of uniqueness cones, and in Theorem 1 we shall obtain a sufficient condition for P-convexity in general cases. In § 3, we shall treat the operators 1)–5)