62. *On the Number of Conjugate Classes of Maximal Subgroups in Finite Groups*

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1. Introduction. M. Numata [1] proved that the nilpotent length of a finite solvable group is at most one plus the number of conjugate classes of the non-normal maximal subgroups.

In this paper we shall prove the following two theorems. One of them partially extends Numata's result.

Theorem 1. *Suppose every non-normal maximal subgroup of a finite group G has the same order. Then G is solvable and the nilpotent length of G is at most two.*

Theorem 2. *The number of conjugate classes of maximal subgroups of a finite non-abelian simple group is at least three.*

Alternating group A_5 has just three conjugate classes of maximal subgroups of it. So the number three in Theorem 2 is best possible. An example related to Theorem 2 is found in the paper [2] due to Goldschmidt, which gives a group-theoretic proof of Burnside's theorem concerning the solvability of groups of order $p^a q^b$ for odd primes p, q. In the paper it is shown that if G is a minimal counter example, then G is simple and the number of conjugate classes of maximal subgroups of G is two. Hence the proof may also be completed by Theorem 2.

2. Proof of the theorems. Let G be a permutation group on Ω, denoted by G^Ω, and H be a subgroup of G. We denote by $I(H)$ a set of the points of Ω left fixed by H. We need the following well-known lemma, which is proved by using Witt's lemma [3, P 20], and Lemma 6 of [4].

Lemma. *Let G be a transitive permutation group on Ω and p be a prime. Suppose P is a p-subgroup of G of maximal order which fixes at least two points. Then $N_G(P)$ is transitive on $I(P)$.*

Proof of Theorem 1. We may suppose that there exists a non-normal maximal subgroup H in G. Let p be a prime dividing $|G:H|$ and let P be a Sylow p-subgroup of G. If $G \gneqq N_G(P)$, then there exists a maximal subgroup L such the $L \geqq N_G(P)$. Since $L \geqq N_G(P)$, we obtain $L = N_G(L)$ and so L is a non-normal maximal subgroup of G. Hence $|L| = |H|$, contrary to our choice of p. Consequently $G \triangleright P$. Let $\bar{L} = L/P$ be any maximal subgroup of $\bar{G} = G/P$. Since p does not divide $|G:L|$,