49. *Examples of Obstructed Holomorphic Maps*

By Takao AKAHORI and Makoto NAMBA
Ryūkyū University and Tohoku University

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1978)

1. By a complex space, we mean a reduced, Hausdorff, complex analytic space. For a complex manifold X, the set $S(X)$ of all compact complex submanifolds of X has a complex space structure (Douady [1]). For $V \in S(X)$, $\dim T_V S(X) \leq \dim H^0(V, \mathcal{O}(N))$, where $T_V S(X)$ is the Zariski tangent space to $S(X)$ at V and $\mathcal{O}(N)$ is the sheaf of sections of the normal bundle N along V (see Namba [4]). We say that V is *unobstructed relative to* X if $S(X)$ is non-singular at V and $\dim T_V S(X) = \dim H^0(V, \mathcal{O}(N))$. Otherwise, V is said to be *obstructed relative to* X. If $H^1(V, \mathcal{O}(N)) = 0$, then V is unobstructed relative to X (Kodaira [2]). Examples of obstructed submanifolds were given by Zappa [5] and Mumford [3].

Now, let V and W be compact complex manifolds. Then, the set $\mathrm{Hol}(V, W)$ of all holomorphic maps of V into W is a complex space. In fact, by identifying $f \in \mathrm{Hol}(V, W)$ with the graph $\Gamma_f \subset V \times W$, $\mathrm{Hol}(V, W)$ is regarded as an open subspace of $S(V \times W)$. Note that the normal bundle along Γ_f is canonically isomorphic to the pull back f^*TW over f of the tangent bundle TW of W. We say that $f \in \mathrm{Hol}(V, W)$ is *unobstructed* if the graph Γ_f is unobstructed relative to $V \times W$. Otherwise, f is said to be *obstructed.* It seems that no example of obstructed holomorphic maps is known. The purpose of this note is to give such examples.

2. Let V be a compact Riemann surface of genus g. Let $\boldsymbol{P}^1$ be the complex projective line. Then $\mathrm{Hol}(V, \boldsymbol{P}^1)$ is nothing but the set of all meromorphic functions on V. It is divided into open (and closed) subspaces:

$$\mathrm{Hol}(V, \boldsymbol{P}^1) = \mathrm{Const} \cup R_1(V) \cup R_2(V) \cup \cdots,$$

where Const is the set of all constant functions and $R_n(V)$ is the set of all meromorphic functions on V of (mapping) order n. If $n \geq g+1$, then $R_n(V)$ is non-empty. If $n \geq g$, then every $f \in R_n(V)$ is unobstructed so that $R_n(V)$ is non-singular and of dimension $2n+1-g$. This follows from the fact that $f^*T\boldsymbol{P}^1 = [2D_\infty(f)]$, where $D_\infty(f)$ is the polar divisor of f and $[2D_\infty(f)]$ is the line bundle determined by the divisor $2D_\infty(f)$.

Theorem. *For* $f \in R_{g-1}(V)$, *assume that* $[2D_\infty(f)] = K_V$ *(the canonical bundle of* V*). Then* f *is unobstructed if and only if*