23. *Boundary Value Problem on Symmetric Homogeneous Spaces*

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1. Introduction. Any eigenfunction of all invariant differential operators on a Riemannian symmetric space can be represented by the Poisson integral of a hyperfunction on its Martin boundary (cf. [2], [3]). We can also formulate a boundary value problem for a little wider class of (not necessarily Riemannian) symmetric spaces. For examples: $SL(n,\boldsymbol{R})/SO(p,n-p)$, $Sp(n,\boldsymbol{R})/U(p,n-p)$, $Sp(n,\boldsymbol{R})/GL(n,\boldsymbol{R})$. Our theorem in this paper is a natural generalization of the result in [2] and [3] under a certain mild condition.

2. Notation. Let G be a connected real semisimple Lie group with finite center, K a maximal compact subgroup of G. Let $\mathfrak{g}$ be the Lie algebra of G, $\mathfrak{k}$ the Lie subalgebra of K in $\mathfrak{g}$. Let θ be the compatible Cartan involution of $\mathfrak{g}$. Let $\mathfrak{a}$ be a maximal abelian subspace in $\mathfrak{p}=\{X\in\mathfrak{g}\,;\,\theta(X)=-X\}$, $\mathfrak{a}^*$ its dual and $\mathfrak{a}_{\mathbf{C}}^*$ the complexification of $\mathfrak{a}^*$. Let Σ be the restricted root system of $(\mathfrak{g},\mathfrak{a})$ and let us introduce an order in Σ. We denote by $\Psi=\{\alpha_1,\cdots,\alpha_l\}$ the set of positive simple roots in this order. Put $\mathfrak{g}^\alpha=\{X\in\mathfrak{g}\,;\,[H,X]=\alpha(H)X$ for any H in $\mathfrak{a}\}$ and let us denote by ρ one-half of the sum of positive roots. Furthermore, let $G=KAN$ be the compatible Iwasawa decomposition, M the centralizer of $\mathfrak{a}$ in K, M^* the normalizer of $\mathfrak{a}$ in K, and $\mathfrak{m}$ and $\mathfrak{n}$ the Lie algebras of M and N, respectively. The quotient group $W=M^*/M$ is called the Weyl group.

3. Preliminary results. We will define the symmetric space G/K_ε where we will investigate simultaneous eigenfunctions of the invariant differential operators.

Definition 1. We call the mapping $\varepsilon:\Sigma\to\{-1,1\}$ *a signature of roots* if the followings are satisfied.

(i) $\varepsilon(\alpha_i)\in\{-1,1\}$ for $\alpha_i\in\Psi$.

(ii) $\varepsilon(\alpha)=\varepsilon(\alpha_1)^{m_1}\cdots\varepsilon(\alpha_l)^{m_l}$ for $\alpha=\sum_{i=1}^{l} m_i\alpha_i\in\Sigma$.

For a given signature ε, we can associate an involutive automorphism θ_ε of $\mathfrak{g}$ by the following:

Definition 2. We define the involutive automorphism θ_ε of $\mathfrak{g}$ so that the conditions

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