13. *Bifurcation of Stable Stationary Solutions from Symmetric Modes*

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Introduction. We consider the following semilinear parabolic system of equations:

$$\begin{aligned} &U_t = D(\sigma)U_{xx} + BU + F(U), \qquad (t,x) \in (0,+\infty)\times(0,L) \\ &U(t,0) = U(t,L) = 0, \end{aligned} \tag{P-1}$$

where $U = {}^t(u(t,x), v(t,x))$, $D(\sigma) = (D_u(\sigma), D_v(\sigma))$ and σ is a real parameter, $B = \begin{pmatrix} a, & b \\ c, & d \end{pmatrix}$ is a real constant matrix and $F(U) = {}^t(f_1(u,v), f_2(u,v))$ is a smooth autonomous nonlinear operator which satisfies

$$F(0) = F_U(0) = 0. \tag{0-1}$$

We assume that B satisfies either of the following conditions:

$$\det B > 0,\ a > 0, \qquad d = 0, \tag{0-2}$$

$$\det B > 0,\ a > 0, \qquad a + d \leqq 0. \tag{0-3}$$

Our main purpose is to show the existence of bifurcation of stable stationary solutions of (P-1) as $D(\sigma)$ varies. Stationary problem of (P-1) and its linearized system of equations at $U=0$ are given as follows:

$$\begin{aligned} &D(\sigma)U_{xx} + BU + F(U) = 0, \\ &U(0) = U(L) = 0, \end{aligned} \tag{P-2}$$

$$\begin{aligned} &D(\sigma)U_{xx} + BU = 0, \\ &U(0) = U(L) = 0. \end{aligned} \tag{P-3}$$

Section 1 deals with the spectrum of (P-3) and the existence of bifurcation of stationary solutions from any mode of the eigenfunction of (P-3) under the appropriate conditions of $D(\sigma)$ and B. Section 2 deals with the asymptotic stability of the bifurcating solutions from symmetric modes. In section 3 we give some examples of biological system to which our theorems can apply.

§ 1. Existence. Using the Fourier series expansion of U,

$$U = \sum_{n=1}^{\infty} U_n \sin\frac{n\pi}{L}x = \sum_{n=1}^{\infty} \begin{pmatrix} u_n \\ v_n \end{pmatrix} \sin\frac{n\pi}{L}x,$$

we obtain the infinite system of linear equations of $\{U_n\}_{n\in N}$:

$$M_nU_n = 0, \quad M_n = \begin{pmatrix} -D_u\left(\dfrac{\pi}{L}\right)^2 n^2 + a, & b \\ c, & -D_v\left(\dfrac{\pi}{L}\right)^2 n^2 + d \end{pmatrix}, \quad n \in \boldsymbol{N}.$$

The roots $\{\alpha_n^i\}_{i=1,2}$ $(\operatorname{Re}\alpha_n^1 \geqq \operatorname{Re}\alpha_n^2)$ of the characteristic equation