7. On the Relations between the Stability of Linear Systems and the Characteristic Roots of the Coefficient Matrix

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1. Introduction. Consider the homogeneous linear differential equation

(1) $$\dot{x}=A(t)x \qquad (x\colon n\text{-vector})$$

where the coefficient $n\times n$ matrix $A(t)$ is continuously differentiable in an interval $I=[0,+\infty)$. In this paper, we shall study the relations between the stability of the system (1) and the characteristic roots of the time-variant coefficient matrix $A(t)$, of which all characteristic roots are constant.

Throughout this paper, we use the vector Euclidean norm $\|x\|=\sqrt{\sum_{i=1}^{n}|x_i|^2}$ and the induced matrix norm $\|A\|=\sup_{\|x\|=1}\|Ax\|$.

Moreover, the definitions of stability, asymptotic stability and instability are the same as given in W. A. Coppel [1].

2. Theorem. We shall give a theorem, which was proved by the second author [3].

Theorem 1. *The homogeneous linear equation*

(1) $$\dot{x}=A(t)x$$

is reduced to the homogeneous linear equation

(2) $$\dot{y}=B(t)y$$

under the transformation

(3) $$x=e^{St}y$$

if and only if there exists a constant matrix S satisfying the equations

(4) $$\dot{A}(t)=SA(t)-A(t)S-e^{St}\cdot\dot{B}(t)\cdot e^{-St}$$

(5) $$A(0)=S+B(0).$$

3. Relations between the stability and the characteristic roots of $A(t)$. In the above Theorem 1, if we can choose a constant matrix B, we can express the fundamental matrix of the system (1) by the form

(6) $$\Phi(t)=e^{St}\cdot e^{Bt}.$$

In this case, the stability of the system (1) is completely decided by the characteristic roots of S and B, therefore is independent of ones of $A(t)$.

Let the coefficient matrix $A(t)$ be given in the following form: