44. Nonlinear Evolution Equations with Variable Domains in Hilbert Spaces

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Let H be a real Hilbert space and denote by $(\cdot,\cdot)$ and $\|\cdot\|$ the inner product and norm in H, respectively. Let ϕ^t be a proper lower semicontinuous convex function on H and put $D_t=\{v\in H;\phi^t(v)<+\infty\}$ and $D(\partial\phi^t)=\{v\in H;\partial\phi^t(v)\neq\emptyset\}$ for each $t\in[0,T]$, where $0<T<+\infty$ and $\partial\phi^t$ is the subdifferential of ϕ^t. In this paper we consider the evolution equation

(E) $$u'(t)+\partial\phi^t(u(t))\ni f(t), \qquad t\in[0,T],$$

where $u'(t)=(d/dt)u(t)$ and f is given in $L^2(0,T;H)$.

In recent years the evolution equation (E) with time-dependent domain $D(\partial\phi^t)$ has been studied by Attouch-Bénilan-Damlamian-Picard [1], Brézis [3], Moreau [7], Kenmochi [5] and Yamada [11]. In the same direction we further study the equation (E).

For each $\lambda>0$ and $t\in[0,T]$, define

$$\phi_\lambda^t(v)=\inf\{\|v-z\|^2/(2\lambda)+\phi^t(z);z\in H\}, \qquad v\in H.$$

According to [4; Chap. II], we see that

$$\partial\phi_\lambda^t(v)=(v-J_\lambda^t v)/\lambda$$

and

$$\phi_\lambda^t(v)=\|v-J_\lambda^t v\|^2/(2\lambda)+\phi^t(J_\lambda^t v)$$

for each $v\in H$, where $J_\lambda^t=(I+\lambda\partial\phi^t)^{-1}$.

Now suppose that

(h1) *there are positive constants α and β such that $\phi^t(z)+\alpha\|z\|+\beta\geqq 0$ for any $t\in[0,T]$ and $z\in H$;*

(h2) *for each $\lambda>0$ and $z\in H$ there is a non-negative function $\rho\in L^1(0,T)$ such that*

$$\phi_\lambda^t(z)-\phi_\lambda^s(z)\leqq\int_s^t\rho(\tau)d\tau$$

for $s,t\in[0,T]$ with $s\leqq t$;

(h3) (i) *for each $r\geqq 0$, there are a number $a_r\in[0,1)$ and functions $b_r,c_r\in L^1(0,T)$ such that $(d/dt)\phi_\lambda^t(z)\leqq a_r\|\partial\phi_\lambda^t(z)\|^2+b_r(t)|\phi_\lambda^t(z)|+c_r(t)$* a.e. *on $[0,T]$ for $z\in H$ with $\|z\|\leqq r$ and $\lambda\in(0,1]$; and* (ii) *there are an H-valued function h on $[0,T]$ and a partition $\{0=t_0<t_1<\cdots<t_N=T\}$ of $[0,T]$ such that $\phi^t(h(t))\in L^1(0,T)$ and the restriction of h to (t_{k-1},t_k) belongs to $W^{1,1}(t_{k-1},t_k;H)$ for $k=1,2,\cdots,N$.*

Theorem. *For each $u_0\in\bar{D}_0$ and $f\in L^2(0,T;H)$ there exists a*