131. *Probability-theoretic Investigations on Inheritance.* *IV_4. Mother-Child Combinations.*

(Further Continuation.)

By Yûsaku KOMATU.

Department of Mathematics, Tokyo Institute of Technology and
Department of Legal Medicine, Tokyo University.

(Comm. by T. FURUHATA, M.J.A., Nov. 12, 1951.)

4. Mother-children combination concerning families with several children.

We have discussed, in the preceding section, the probabilities of mother-children combinations concerning families with two children. The results can be further generalized to several children case. We now consider the set of a mother and her n children produced from a common father, n being arbitrary but fixed.

Consider again an inherited character consisting of m genes A_i $(i=1,\dots,m)$ with distribution-probability $\{p_i\}$, the distribution being here also supposed to be in an equilibrium state. In general, the number of permutations, admitting the repetition, of selecting any n types of children without kinship is equal to

$$(4.1) \qquad 2^{-n}m^n(m+1)^n.$$

But, if the children are restricted such that they have a common mother, then the corresponding number becomes

$$(4.2) \qquad m^n \quad \text{or} \quad (2m-1)^n$$

according to the mother of a homozygote or of a heterozygote, respectively. If they are further restricted such as to have a father also in common, then number of possible permutations reduces to a very small one. In fact, corresponding to that in §3 of IV, we get the following table.

Mating	$A_{ii}\times A_{ii}$	$A_{ii}\times A_{ik}$	$A_{ii}\times A_{hh}$	$A_{ii}\times A_{hk}$	$A_{ij}\times A_{ij}$	$A_{ij}\times A_{ik}$	$A_{ij}\times A_{hk}$
Permutation	1	2^n	1	2^n	3^n	4^n	4^n

Making use of a table on one-child case written in §3 of IV, we can easily construct the corresponding table on n-children case.

We denote by $\pi(A_{ij}; A_{h_1k_1},\dots,A_{h_nk_n})$ or briefly by

$$(4.3) \qquad \pi(ij; h_1k_1,\dots,h_nk_n) \quad (i, j, h_\nu, k_\nu=1,\dots,m;\ \nu=1,\dots,n)$$

the probability of appearing of a combination of a mother A_{ij} and her n children among which νth child is of type $A_{h_\nu k_\nu}$ for $\nu=1,\dots,n$. This quantity is, as before, equal to zero provided either of n relations holds: