88. On the Adiabatic Theorem for the Hamiltonian System of Differential Equations in the Classical Mechanics. I

By Takashi KASUGA

Department of Mathematics, University of Osaka

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Introduction. In this paper, we shall use the following abbreviations:

$$p=(p_1,\cdots,p_n),\ q=(q_1,\cdots,q_n),\ (p,q)=(p_1,\cdots,p_n,q_1,\cdots,q_n)$$
$$dp/dt=(dp_1/dt,\cdots,dp_n/dt),\ dq/dt=(dq_1/dt,\cdots,dq_n/dt),$$
$$\partial H/\partial p=(\partial H/\partial p_1,\cdots,\partial H/\partial p_n),\ \partial H/\partial q=(\partial H/\partial q_1,\cdots,\partial H/\partial q_n)$$
$$dpdq=dp_1\cdots dp_n\cdot dq_1\cdots dq_n.$$

Let $H(p,q,s)$ be a Hamiltonian containing a parameter s $(1\geqq s\geqq 0)$. Roughly speaking, a quantity $I(p,q,s)$ which is a global integral of the system

$$(1)\qquad dp/dt=-\partial H/\partial q(p,q,s),\ dq/dt=\partial H/\partial p(p,q,s)$$

for every fixed s $(1\geqq s\geqq 0)$, is called an adiabatic invariant of (1), if $I(p,q,t/\lambda)$ is conserved along all (nearly all)[1] trajectories of the following system

$$(2)\qquad dp/dt=-\partial H/\partial q(p,q,t/\lambda),\ dq/dt=\partial H/\partial p(p,q,t/\lambda)$$

in the whole interval of time $0\leqq t\leqq\lambda$, asymptotically for $\lambda\to+\infty$. The fundamental case of the adiabatic theorem in the classical mechanics is the case where (1) has no one-valued Lebesgue measurable global integral other than the functions of the energy integral for almost all s $(1\geqq s\geqq 0)$.[2] In this case, the phase volume

$$\mathfrak{J}(p,q,s)=\int_{I_{E,s}} dpdq$$

where $I_{E,s}$ means the domain in (p,q)-space enclosed by the energy surface $S_{E,s}=\{(p,q)\mid H(p,q,s)=E\}$ of (1) passing through (p,q), is one (and essentially the only one)[3] adiabatic invariant. In the following, we shall call this proposition the adiabatic theorem.

The adiabatic theorem plays an important rôle not only in the statistical mechanics, but also in various other branches of physics. But as far as we know, satisfactory proofs of the theorem exist

1) Here the word "nearly all" is used in a vague sense.

2) The cases where this assumption does not hold, can be reduced to this fundamental case if the reductions as given in T. Levi-civita [7] are possible and the reduced system satisfies an assumption similar to this assumption. Cf. T. Levi-civita [7], also H. Geppert [1].

3) The "only one" part of the adiabatic theorem shall be treated in Part IV of this paper.