103. *Probaility-theoretic Investigations on Inheritance.*
III$_2$. Further Discussions on Cross-Breeding.
(Continuation.)

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3. Proof of main result.

In order to prove the main result stated in (2.30), (2.31) by induction, we must first generalize the table inserted in the preceding section to the case of passage from the $(n-1)$th to the nth generation, stating as follows. In the table, the notation $\mu^{(l)}_{uv}$ is used in slightly exteded meaning such that a convention is made:

$$\mu^{(l)}_{uv}=\mu^{(l)}_{vu},\quad \mu^{(l)}_{uu}=0\quad (u, v=0, 1, 2, \dots, 2^{l-1}). \tag{3.1}$$

Further, the notation $d(\alpha)$ for any natural number α means the highest power of 2 which appears in the decomposition of 2α into prime numbers; i.e., $d(\alpha)$ is defined in such a manner that α is divisible by $2^{d(\alpha)-1}$ but not by $2^{d(\alpha)}$. In particular, if α is an odd number, $d(\alpha)$ is always equal to unity.

class in the nth generation	mating-class in the $(n-1)$th generation	frequency of each mating-class
X'^{2^n}	$X'^{2^{n-1}}\times X'^{2^{n-1}}$	$\lambda'-\sum_{l=1}^{n}\sum_{k=1}^{2^{l-1}}\mu_{0k}$
$X'^{2^n-(2\alpha-1)}X''^{2\alpha-1}$ $(1\leqq\alpha\leqq 2^{n-2})$	$X'^{2^{n-1}-h}X''^{h}\times X'^{2^{n-1}-(2\alpha-1)+h}X''^{2\alpha-1-h}$ $(0\leqq h<\alpha)$	$2\mu^{(n)}_{h,2\alpha-1-h}$ $2\mu^{(n)}_{h,2\alpha-h}$
$X'^{2^n-2\alpha}X''^{2\alpha}$ $(1\leqq\alpha\leqq 2^{n-2})$	$\begin{cases}X'^{2^{n-1}-h}X''^{h}\times X'^{2^{n-1}-2\alpha+h}X''^{2\alpha-h}\ (0\leqq h<\alpha)\\ X'^{2^{n-1}-\alpha}X''^{\alpha}\times X'^{2^{n-1}-\alpha}X''^{\alpha}\end{cases}$	$\sum_{l=n+1-d(\alpha)}^{n}\Big(2\sum_{0\leqq h<2^{l-n-1}\alpha}\mu^{(l-1)}_{h,2^{l-n}\alpha-h}-\sum_{k=0}^{2^{l-1}}\mu^{(l)}_{2^{l-n}\alpha,k}\Big)\,2\mu^{(n)}_{h,2\alpha-h}$
$X'^{2^n-2\alpha}X''^{2\alpha}$ $(2^{n-2}<\alpha<2^{n-1})$	$\begin{cases}X'^{2^{n-1}-h}X''^{h}\times X'^{2^{n-1}-2\alpha+h}X''^{2\alpha-h}\ (\alpha<h\leqq 2^{n-1})\\ X'^{2^{n-1}-\alpha}X''^{\alpha}\times X'^{2^{n-1}-\alpha}X''^{\alpha}\end{cases}$	$\sum_{l=n+1-d(\alpha)}^{n}\Big(2\sum_{2^{l-2}\geqq h>2^{l-n-1}\alpha}\mu^{(l-1)}_{h,2^{l-n}\alpha-h}-\sum_{k=0}^{2^{l-1}}\mu^{(l)}_{2^{l-n}\alpha,k}\Big)\,2\mu^{(n)}_{h,2\alpha-1-h}$
$X'^{2^n-(2\alpha-1)}X''^{2\alpha-1}$ $(2^{n-2}<\alpha\leqq 2^{n-1})$	$X'^{2^{n-1}-h}X''^{h}\times X'^{2^{n-1}-(2\alpha-1)+h}X''^{2\alpha-1-h}$ $(\alpha\leqq h\leqq 2^{n-1})$	
X''^{2^n}	$X''^{2^{n-1}}\times X''^{2^{n-1}}$	$\lambda''-\sum_{l=1}^{n}\sum_{k=0}^{2^{l-1}-1}\mu^{(l)}_{2^{l-1},k}$
		$\lambda'+\lambda''=1$

In order to assert the validity of the table by induction, we go back by one generation and assume now the validity of the corresponding table for $(n-1)$th generation.

First, there exist $2^{n-1}+1$ possible types of matings concerned by the class $X'^{2^{n-1}}$ in the $(n-1)$th generation; they are expressed