112. *On an Example of a Measure Preserving Transformation Which is Not Conjugate to Its Inverse.*

By Hirotada ANZAI.

(Comm. by K. KUNUGI, M.J.A., Nov. 12, 1951.)

1. **Introduction.** If G is a symmetric group it is evident that any element T of G is conjugate to its inverse T^{-1}, because T and T^{-1} are decomposed into the same types of cycles. If the group G of all measure preserving transformations on a measure space of Lebesgue type is regarded as a generalization of symmetric groups there arises naturally the question whether any measure preserving transformation is conjugate to its inverse or not. If the measure preserving transformation under consideration is ergodic and has pure point spectrum, it was shown by J. von Neumann that the invariant for its conjugate class was the additive group of the eigenvalues.[1] Therefore for such a transformation the question is answered affirmatively.[2] But generally the question has remained open, because any general invariants for the conjugate classes have not yet been found. We obtained a criterion to be conjugate to each other for some ergodic transformations of special types in our previous paper "Ergodic Skew Product Transformations on the Torus".[3] By using a similar criterion to this and by the same method as the one in §7 of E.S. we shall give an example of a measure preserving transformation T which is not conjugate to its inverse. In this sense this paper is a continuation of the previous paper E.S..

We shall not evade to repeat some of the definitions and the results which appeared already in E.S.

2. **The criterion that T is conjugate to T^{-1}.** Let X and Y be the usual Lebesgue measure spaces of the circles with unit length. We denote by m the Lebesgue measure on X. The direct product measure space Ω of X and Y is the usual Lebesgue measure space of the two-dimensional torus. Let γ be an irrational number on X and let $\alpha(x)$ be a measurable mapping from X into Y. Then the transformation T on Ω which is defined in the following way is called a skew product measure preserving transformation:

$$T(x,\ y) = (x+\gamma,\ y+\alpha(x)).$$

$\alpha(x)$ is called the α-function of the skew product transformation T.

1) "Zur Operatorenmethode in der klassischen Mechanik" Ann. of Math. **33** (1932).

2) P.R. Halmos and J. von Neumann: Operator methods in classical mechanics II, Ann. of Math. **43** (1942).

3) Osaka Math. J. Vol. 3, No. 1 (1951). We shall refer to this paper as E.S.