111. On Spaces with a Complete Structure.

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The purpose of this note is to study the problem: Is it true that every completely regular space with a complete structure is homeomorphic to a closed subset of a Cartesian product of the space of real numbers with its usual topology?

Concerning the above problem, under a restriction with respect to cardinal numbers of the spaces, an affirmative answer will be given in this note.

§ 1. Definition 1.[1] Let us call the structure of a completely regular space X with the uniformity made up of all countable normal coverings of the space X the *e-structure* of X and denote by eX. Moreover we say the space with the complete e-structure to be *e-complete* and let us call a cardinal number $\mathfrak{m}$ to be *e-complete* if the discrete space with the potency $\mathfrak{m}$ is e-complete.

Definition 2.[2] Let X be a completely regular space and let $C(X, R)$ be the set of all real-valued continuous functions with domain X. Further-more let f be a function in $C(X, R)$. Then the set of points in X for which f vanishes is said to be a *Z-set* and is denoted by $Z(f)$. Finally let $Z(X)$ be the family of all Z-sets of X. Then a subfamily $\mathfrak{A}$, of the family $Z(X)$ is said to be a *CZ-maximal family* of X if $\mathfrak{A}$ enjoys the following four conditions:

a) $\mathfrak{A}$ is not an empty family,

b) $\mathfrak{A}$ does not contain a void set,

c) $\mathfrak{A}$ never contains countable subfamilies with total intersection void, and

d) $\mathfrak{A}$ is maximal with respect to the properties a), b) and c).

§ 2. **Lemma 1.** *Every CZ-maximal family of a completely regular space X is a Cauchy family of the e-structure eX. For any Cauchy family of the e-structure eX there exists a Cauchy family such that they are equivalent.*

Lemma 2.[3] *A completely regular space is homeomorphic to a closed subset of a Cartesian product of the reals if and only if for any CZ-maximal family the total intersection is not void.*

1) J. W. Tukey: Convergence and uniformity in topology, Princeton University press, Princeton 1940; T. Shirota: On systems of structures of a completely regular space, Osaka Math. J. **2** (1950).

2) E. Hewitt: Rings of real valued continuous functions, Trans. Amer. Math. Soc. **64** (1948).

3) E. Hewitt: loc. cit, 2).