100. *Probability-theoretic Investigations on Inheritance. XIII$_2$. Estimation of Genotypes.*

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(Comm. by T. FURUHATA, M.J.A., Oct. 13, 1952.)

3. Estimation without reference to spouse.

The problem discussed in §2 concerned the case where the type of a spouse of an individual in question is also taken into account. The corresponding problem may be treated independently of the type of a spouse.

We first consider again the simplest case, *the Q blood type*. Let an individual of phenotype Q be given. Then, the type q of its child is impossible unless the individual is heterozygotic. Hence, we have only to consider the case where all the n children of the individual are of the type Q. In this case, we denote by

$$\Pr\{Q=QQ|\rightarrow Q^n\} \quad \text{and} \quad \Pr\{Q=Qq|\rightarrow Q^n\}$$

the probabilities a posteriori of the individual to be of homozygote QQ and of heterozygote Qq, respectively, which will be determined in the following lines.

Now, the probabilities a priori of QQ and Qq among Q are regarded as $\overline{QQ}/\overline{Q}=u/(1+v)$ and $\overline{Qq}/\overline{Q}=2v/(1+v)$, respectively, the ratio being $u:2v$. An individual QQ produces Q alone, while an individual Qq produces Q with probability

$$\frac{\pi(Qq;QQ)+\pi(Qq;Qq)}{\overline{Qq}}=\frac{1+u}{2},$$

the π's denoting the probabilities of mother-child combinations defined in §1 of IV, which may also be regarded as those of father-child combinations. Thus, based on the Bayes' theorem, we get the desired probabilities

$$(3.1) \qquad \Pr\{Q=QQ|\rightarrow Q^n\}=\frac{u\cdot 1^n}{u\cdot 1^n+2v\left(\dfrac{1+u}{2}\right)^n}=\frac{2^{n-1}u}{2^{n-1}u+v(1+u)^n},$$

$$(3.1') \qquad \Pr\{Q=Qq|\rightarrow Q^n\}=1-\Pr\{Q=QQ|\rightarrow Q^n\}=\frac{v(1+u)^n}{2^{n-1}u+v(1+u)^n}.$$

We proceed to deal with *the ABO blood type*. Let an individual of phenotype A be given. If it is homozygotic, then the type of a child is restricted to A or AB, while if it is heterozygotic, then any type of a child is possible. Accordingly, if there exists at least one