90. *On Cauchy's Problem in the Large for Wave Equations.*

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(Comm. by Z. SUETUNA, M.J.A., Oct. 13, 1952.)

§1. *Introduction.* Let R be a connected domain of an orientable, m-dimensional Riemannian space with the metric $ds^2=g_{ij}(x)dx^i dx^j$. We consider the wave equation

$$(1.1)\qquad \frac{\partial^2 u(x,t)}{\partial t^2}=A_x u(x,t),\ -\infty<t<\infty,$$

with Cauchy's data

$$(1.2)\qquad u(x,0)=f(x),\quad \frac{\partial u(x,0)}{\partial t}=h(x).$$

Here the differential operator $A=A_x$ defined by

$$(1.3)\qquad A_x f(x)=b^{ij}(x)\frac{\partial^2 f(x)}{\partial x^i \partial x^j}+a^i(x)\frac{\partial f(x)}{\partial x^i}+e(x)f(x)$$

is *elliptic* in the sense that the quadratic form $b^{ij}(x)\xi_i\xi_j$ is >0 for $\sum_i(\xi_i)^2>0$. Since the value of $A_x f(x)$ must be independent of the local coordinates $(x^1,\dots,x^m)$ of the point x, the coefficients $a^i(x)$ and $b^{ij}(x)$ must be transformed, by the coordinates change $x\to\bar{x}$, respectively into

$$(1.4)\qquad \bar{a}^i(\bar{x})=\frac{\partial \bar{x}^i}{\partial x^k}a^k(x)+\frac{\partial^2 \bar{x}^i}{\partial x^k\partial x^s}b^{ks}(x) \text{ and } \bar{b}^{ij}(\bar{x})=\frac{\partial \bar{x}^i}{\partial x^k}\frac{\partial \bar{x}^j}{\partial x^s}b^{ks}(x).$$

For the sake of simplicity, we assume that $g_{ij}(x)$, $b^{ij}(x)$, $a^i(x)$ and $e(x)$ are infinitely differentiable functions of the local coordinates $(x^1,\dots,x^m)$.

Since we are concerned with *the existence in the large of the integral* of (1.1)–(1.2), it will perhaps be necessary to rely upon operator-theoretical method[1]. We here assume that the operator A_x is, as in the case of Laplacian, *formally self-adjoint* and *non-positive definite*, viz.

$$(1.5)\qquad \int_R (A_x f(x))h(x)dx=\int_R f(x)(A_x h(x))dx \text{ and } \int_R (A_x f(x))f(x)dx\leqq 0$$
$$(dx=\sqrt{g(x)}\,dx^1\dots dx^m,\quad g(x)=det(g_{ij}(x)),$$

if $f(x)$ and $h(x)$ are twice continuously differentiable such that $f(x)$ vanishes outside a compact set contained in the interior of R. Then we may integrate, by virtue of the Hilbert space technique, an operator-theoretical variant of (1.1)–(1.2) It will next be shown, by a parametrix consideration, that this operator-theoretical integral is, for sufficiently smooth initial data (1.2), equivalent to the ordinary integral of the genuine differential equation (1.1)–(1.2). It is