118. Probability-theoretic Investigations on Inheritance. XV$_4$. Detection of Interchange of Infants.

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(Comm. by T. FURUHATA, M.J.A., Nov. 12, 1952.)

7. Illustrative examples, recessive genes being existent.

Problems and results discussed in the preceding sections have exclusively concerned genotypes. In case of existence of recessive genes, the procedure has only to be modified according to the corresponding dominance relations. We give here, as illustrative examples, the results on ABO, Q as well as $Qq_{\pm}$ blood types.

First, for *ABO blood type*, applying the process explained in § 3 to a corresponding table, we get

$$
\begin{aligned}
&G_0(O, O) && && =r^4(1-r^2),\\
&G_0(O, A) && =G_0(A, O) && =pqr^2(p+2r)(2-q),\\
&G_0(O, B) && =G_0(B, O) && =pqr^2(q+2r)(2-p),\\
&G_0(O, AB) && =G_0(AB, O) && =2pqr^2(r^2+2pq),\\
&G_0(A, A) && && =p^2q(p+2r)^2(2-q),\\
&G_0(A, B) && =G_0(B, A) && =0,\\
&G_0(A, AB) && =G_0(AB, A) && =2p^2qr^2(p+2r),\\
&G_0(B, B) && && =pq^2(q+2r)^2(2-p),\\
&G_0(B, AB) && =G_0(AB, B) && =2pq^2r^2(q+2r),\\
&G_0(AB, AB) && && =4p^2q^2r^2.
\end{aligned}
\tag{7.1}
$$

The total sum of these sixteen quantities represents the probability G_{0ABO} of detecting the interchange of infants within the first triple. The expression for this probability being evidently symmetric with respect to p and q, it can be expressed in a unique manner as a function of two independent variables r and pq. In fact, by remembering a recurrence formula

$$p^\nu+q^\nu=(1-r)(p^{\nu-1}+q^{\nu-1})-pq(p^{\nu-2}+q^{\nu-2}),$$

we obtain an expression

$$
\begin{aligned}
G_{0ABO}=r^4(1-r^2)+2pq(1+r+3r^2+3r^3+2r^4)\\
-p^2q^2(7+12r+5r^2)+2p^3q^3.
\end{aligned}
\tag{7.2}
$$

Next, we obtain in turn