111. *On Right-Regular-Ideal-Rings* *)

By Hisao TOMINAGA
Department of Mathematics, Okayama University
(Comm. by Z. SUETUNA, M.J.A., Nov. 12, 1953)

1. In his paper**) 4) T. Nakayama defined the notion of regularity of modules, which played an important rôle in his Galois theory. In this note we consider a ring in which every non-zero right ideal is right-regular and we call such a ring a right-regular-ideal-ring. To be easily seen, the notion of right-regular-ideal-rings is a generalization of that of simple rings as well as principal-right-ideal-domains***).

Throughout this paper, except in the last remark, the term "ring" will mean a non-zero ring with an identity, and K will signify a ring. The notation $\cong$ will be used to denote a K-isomorphism between two K-right-modules, unless otherwise specified. Further by minimum and maximum conditions in rings we shall understand those which are related to the right ideals.

Let M be a K-module. If the identity element of K operates as the identity operator for M, then M is called *unitary*. And if a finite generating system $\{u_1, \dots, u_n\}$ of a unitary K-module M is such that $\sum_{i=1}^{n} u_i k_i = 0\ (k_i \in K)$ implies $k_i = 0\ (i = 1, \dots, n)$, then we call it an *independent K-basis* of M.

Let M be a unitary K-module, then we shall denote by M^n the direct sum of its n copies written as column vectors. Thus $M \cong K^m$ means that M has an independent K-basis of m elements. On the other hand, we shall denote by ${}^nM$ the direct sum of its n copies written as row vectors. Naturally, ${}^nM$ may be considered as a K_n-module, where K_n denotes the total $n \times n$ matrix ring over K. Hereafter, let ${}^nM$ stand for the K_n-module with the natural K_n-module structure. To be easily verified, $({}^pM)^q$ is K_p-isomorphic to ${}^p(M^q)$, where p, q are natural numbers. From this fact, we can use the notation ${}^pM^q$ instead of ${}^p(M^q)$ or $({}^pM)^q$.

2. A non-zero unitary K-module M is said to be *right-regular* with respect to K if there exist two natural numbers p, q such that $M^p \cong K^q$. And a ring K is called a *right-regular-ideal-ring*

---

*) I wish to thank Prof. G. Azumaya for his useful advices given to me.

**) Numbers in brackets refer to the references at the end of this paper.

***) Throughout the paper, a simple ring means a total matrix ring over a division ring. And a principal-right-ideal-domain means an integral domain in which every right ideal is principal.