129. On the Propagation of Regularity of Solutions of Partial Differential Equations with Constant Coefficients

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1. Let $P\left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}, \cdots, \dfrac{\partial}{\partial x_n}\right)$ be a partial differential operator of order m with constant coefficients. Let ξ be a unit vector of the dual space $\varXi^n$ of $R^n = \{(x_1, x_2, \cdots, x_n)\}$ and for any vector ξ, $S(\xi, h)$ the spherical neighbourhood of ξ with radius h. Then we define the ξ-regularity of P as follows:

Definition. $P\left(\dfrac{\partial}{\partial x}\right)$ is ξ-regular if every distribution solution u of the equation $Pu = 0$ defined in $S(0, h)$ for some h, is in $C^0(S(0, l))$ for some l, whenever u belongs to $C^p(S(0, h) \frown \{x \mid (x, \xi) \leqq 0\})$, where $l(<h)$ and p are independent of u.

In the present note we give some characterization of the ξ-regularity using A. Seidenberg's Theorem [1] as follows:

Theorem. *The necessary and sufficient condition for P to be ξ-regular is the following: there are a neighbourhood $S(\xi, \delta)$, positive numbers A, B, L, α such that if for any real number s, for any real vector $\eta \in \varXi^n$ and for any $\xi' \in S(\xi, \delta)$*

$$A < s < B(|\eta| + 1)^\alpha \quad and \quad |\eta| > L,$$

then $s\xi' + i\eta$ does not satisfy the characteristic equation of P, i.e.,

$$P(s\xi' + i\eta) \neq 0.$$

By Theorem and using Hörmander's considerations [2] we see the following

Corollary 1. *If P is homogeneous and Q is weaker than P and of order $< m$, then $P + Q$ is ξ-regular, whenever P is so.*

Corollary 2. *Let $n \geqq 3$. Then the following conditions are equivalent:*

(1) *$P + Q$ is ξ-regular for any Q such that the order of $P >$ the order of Q,*

(2) *$P(\xi) \neq 0$ and if a real $\eta(\neq 0)$ satisfies the equation*

$$P(\eta) = 0,$$

then

$$(\xi, (grad\ P))\ (\eta) \neq 0, \quad and$$

(3) *P is of principal type and is hypo-ξ-regular.*

Corollary 3. *If P is not hypo-elliptic, then there exists an ξ*