74. Quasi-Conformal Extension of Meier's Theorem

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We shall show that the so-called Meier's topological analogue of Plessner's theorem ([5], Satz 5, cf. [2], p. 154) is true of quasi-conformal functions in $U:|z|<1$. A function $w=f(z)$ defined in a plane domain G with its values in the w-sphere $\Omega:|w|\leqq\infty$ is called quasi-conformal (precisely, K-quasi-conformal) in G if f is of the composed form: $g\circ T(z)$, where $\zeta=T(z)$ is a K-quasi-conformal homeomorphism from G onto another plane domain G' and $w=g(\zeta)$ is meromorphic in G' (cf. [4], p. 250).

Let $f(z)$ be a quasi-conformal function in U and let $e^{i\theta}$ be a point of $\Gamma:|z|=1$. Then the cluster set $C(f, e^{i\theta})$, an angular cluster set $C_\Delta(f, e^{i\theta})$ and a chordal cluster set $C_{\rho(\varphi)}(f, e^{i\theta})$ are defined in the same manner as in [2] (pp. 1, 73 and 72), where Δ is the interior of a triangle in U with one vertex $e^{i\theta}$ (simply, "angle Δ at $e^{i\theta}$") and $\rho(\varphi)$ is a chord of Γ passing through $e^{i\theta}$ and making a directed angle φ, $|\varphi|<\pi/2$, with the radius to $e^{i\theta}$. A point $e^{i\theta}\in\Gamma$ is a Plessner point of f if $C_\Delta(f, e^{i\theta})=\Omega$ for any angle Δ at $e^{i\theta}$. A point $e^{i\theta}\in\Gamma$ is a Meier point of f if $C(f, e^{i\theta})\neq\Omega$ and $C_{\rho(\varphi)}(f, e^{i\theta})=C(f, e^{i\theta})$ for all φ, $|\varphi|<\pi/2$. We denote by $I(f)$ ($M(f)$, resp.) the set of all Plessner points (Meier points, resp.) of f.

We first prove a topological analogue of Fatou's theorem (cf. [5], Satz 6, [2], p. 154).

Theorem 1. *Let f be a bounded quasi-conformal function in U. Then $\Gamma\backslash M(f)$ is of first Baire category on Γ.*

Proof. We shall use the Schwarz lemma for quasi-conformal functions (cf. [3]) in the following form: Let $h(z)$ be a K-quasi-conformal function in the disk $\delta(z_0, q):|z-z_0|<q$. If $|h(z)|<M, M>0$ being a constant, in $\delta(z_0, q)$, then

$$(1)\qquad |h(z)-h(z_0)|\leqq 8Mq^{-1/K}|z-z_0|^{1/K},\qquad z\in\delta(z_0, q).$$

We let, for the proof, $h(z)=g\circ T(z)$, where T is a K-quasi-conformal self-homeomorphism of $\delta(z_0, q)$ with $z_0=T(z_0)$, which we may suppose, and g is holomorphic in $\delta(z_0, q)$. Then, Theorem 5, (9) of Mori [6] reads

$$|T(z)-T(z_0)|\leqq 4q^{1-(1/K)}|z-z_0|^{1/K}.$$

Combined with the Schwarz lemma for the bounded g, this gives (1).