178. *On the Adjoint Semigroups of Rings. I*

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(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1970)

The fundamental notions, used in this paper, can be found in A. H. Clifford-G. B. Preston [1], N. Jacobson [3] and N. H. McCoy [4]. As it is well known, H. J. Hoehnke [2] has developed an interesting theory of a radical for semigroups with zero such that this radical is similar in certain sense to the Jacobson radical of rings (see [3]). Furthermore, H. Seidel [5] has introduced for semigroups, not necessarily having zero, the concept of right quasiregular elements, and he has proved [5] with the help of right quasiregular elements, that the Hoehnke radical of any semigroup S with zero coincides with the nil radical of S. (In addition see yet author's paper [7] on six further similar radicals of semigroups.) We recall, that an element $s \in S$ is right quasiregular in the semigroup S if and only if for arbitrary elements $t \in S$ and $u \in S$ nonnegative rational integers m and n there exist such that $s^m t = s^n u$ holds, eventually $s^0 t$ denoting the element t.

Some results on general radicals of semigroups with zero are discussed in author's paper [8].

A semigroup S having twosided unity element e is said to be *almost right quasiregular*, if $S = e \cup Q$ with $e \notin Q$ holds such that Q is a subsemigroup of S, and any element of Q is right quasiregular in S.

Furthermore, a semigroup S with both twosided unity element e and zero will be called almost nil (or almost nilpotent), if $S = e \cup N$ with $e \notin N$ holds, where N is a nil (or nilpotent, respectively) subsemigroup of S. Here N is said to have a bounded index m of nilpotency of elements, if a natural number m there exists such that $x^m = 0 \in N$ for any $x \in N$ holds.

Thirdly, a semigroup S with both twosided unity element e and zero will be called almost trivial, if $S = e \cup T$ with $e \notin T$ holds, where T is a subsemigroup of S such that all products xy for arbitrary $x \in T$ and $y \in T$ coincide with the zero element of S.

Therefore, any almost trivial semigroup is commutative.

All rings, considered here, will be associative. For any ring A, the elements of A form with respect to the circle operation $a \circ b = a + b - ab$ a semigroup S, which is called the adjoint semigroup of the ring A. Obviously the zero 0 of A is the twosided unity element of S. Furthermore an element e of A is a right zero of S if and only if e is